A COMPARISON OF BUFFER STRIP AND NON-BUFFER STRIP JOINT DESIGNS

James Michael Gill

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

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by

James Michael Gill

June 1976

Thesis Advisor: M.H. Bank II

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE | READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)
   A COMPARISON OF BUFFER STRIP AND NON-BUFFER STRIP JOINT DESIGNS
5. TYPE OF REPORT & PERIOD COVERED
   ENGINEER'S THESIS; (June 1976)
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)
   JAMES MICHAEL GILL
8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS
   Naval Postgraduate School
   Monterey, California 93940
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS
   Naval Postgraduate School
   Monterey, California 93940
12. REPORT DATE
   June 1976
13. NUMBER OF PAGES
   162
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
   Naval Postgraduate School
   Monterey, California 93940
15. SECURITY CLASS. (of this report)
   Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)
   Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
   Bolted Joints
   Composite Materials
   Computer Program
   Epoxy Resins
   Finite Element Analysis
   Graphite
   Graphite-Epoxy Laminates
   Laminates
   Plates
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20.

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by

James Michael Gill
Lieutenant Commander, United States Navy
B.Ae., University of Minnesota, 1965
M.S., Naval Postgraduate School, 1975

Submitted in partial fulfillment of the
requirements for the degree of

AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL

June 1976

ABSTRACT

Buffer strip and non-buffer strip bolted wing skin type joints made from NARMCO 5208/T300 graphite-epoxy material were designed, and the excess bearing capacity and weight of these joints were calculated for a wide range of laminate compositions, bolt hole sizes, and number of bolt holes. Design load conditions representative of an advanced fighter type aircraft were chosen. Joint designs were arbitrarily restrained by assumed manufacturing conditions, assumed interface conditions, and imposed laminate composition restrictions. Charts were prepared from which relative joint efficiencies could be determined but no attempt was made to analyze the effect of the arbitrary design restrictions. The advantages and penalties for buffer strip design were discussed and recommendations for future studies were made.

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LIST OF SYMBOLS

AD = overall buffer strip joint width (in.)

A_n = constant of proportionality between the bolt load P and the portion of P reacted each of the primary strips of a buffer strip joint

B = excess bearing capacity

D = bolt hole diameter (in.)

E_x = tensile modulus (lbf./in.^{2})

E_{x_1} = tensile modulus of the primary strips in a buffer strip joint (lbf./in.^{2})

E_{x_2} = tensile modulus of the buffer strip material in a buffer strip joint (lbf./in.^{2})

F_{BP} = bypass force (lbf.)

F_S = shear load passing P_1 from a buffer strip to a primary strip (lbf.)

F_r = reaction force (lbf.)

$f(\frac{a}{r})^i_\alpha$ = effective isotropic stress concentration factor at location i for load condition α

f_s = shear stress between the buffer and primary strips of a buffer strip joint (lbf./in.^{2})

i = indicator of exact position on the hole

L = tensile load (lbf.)

ℓD = side length (in.)

M = number of rows of bolts in a buffer strip joint

m = reaction moment (in.-lbf.)

N = number of rows of bolts in a non-buffer strip joint

N_x = tensile load (lbf./in.)

N_{xy} = shear load (lbf./in.)

P = tensile bolt load (lbf.)

P_s = shear bolt load (lbf.)

P_1 = portion of P reacted in each of the primary strips of a buffer strip joint (lbf.)

P_2 = portion of P reacted in the buffer strip material of a buffer strip joint (lbf.)

R = resultant bolt load (lbf.)

S_1 = representative applied stress (lbf./in.^{2})

S_2 = representative failure stress (lbf./in.^{2})

t = plate thickness (in.)

t^* = effective plate thickness (in.)

$t_{\pm 45}$ = total thickness of $\pm 45^o$ laminae (in.)

$W_B D$ = width of buffer strip material (in.)

$W_1 D$ = half-width of primary laminate material in a buffer strip joint (in.)

Z = percentage of zero degree plies in the primary strips of a buffer strip joint

α = subscript denoting applied load conditions as follows:

= bx   bearing in x direction

= by   bearing in y direction

= tx   tension in x direction

= ty   tension in y direction

= xy   shear

ϵ = strain (in./in.)

λ_{α}^{i} = finite width correction factor at location i for

applied load condition α

η = indicator of position in a joint

σ_{BR} = resultant bolt bearing stress (lbf./in.^{2})

σ^i = net tangential stress at location i

σ_α = applied stress for load condition α

ACKNOWLEDGEMENT

The author is particularly grateful to Mr. J.R. Eisenmann of Fort Worth, Texas whose work laid much of the foundation for this study. He also wishes to offer his thanks to Assistant Professor M.H. Bank II of the Naval Postgraduate School for his help in organizing and writing this report. The author also wishes to acknowledge Candida Gill whose patience and understanding during this investigation was so needed.

I. INTRODUCTION

A. BACKGROUND

Aerospace structural design requirements have often been characterized by a demand for high strength and low weight. In many cases, the design possibilities have been limited by the available manufacturing technology and the acceptable manufacturing costs. The most successful designs were usually those which met all these constraints most efficiently.

The demand for high structural efficiency led to the development of advanced laminated composites which have higher strength-to-weight ratios and better fatigue properties than conventional structural materials (Ref. 1). As discussed in Ref. 2, it has also been possible to build laminated composite materials with higher modulus values than those characteristic of conventional materials. Accurate methods for tailoring the properties of such materials have evolved and such tailoring has become accepted design practice (Refs. 3,4,5). Waddoups, in Ref. 6, explained that significant gains in structural efficiency have been demonstrated by doing no more than substituting such a tailored composite for a conventional material keeping the geometry and mating interfaces the same.

These gains were realized because the composite materials used were less dense than the conventional

structural materials which they replaced. These material substitutions did not exploit the high strength and high modulus values achievable with advanced laminated composites.

B. OBJECTIVES

This study was intended to demonstrate the structural efficiencies which could be achieved by designs which took into account some of the high strength and high modulus properties of advanced composites. It was decided to demonstrate these efficiencies by analyzing the behavior of a plate of laminated material in the region of a bolted joint. This situation corresponded to that of a wing skin attached by a bolted fitting to the fuselage of an aircraft.

It was assumed that the wing skin would be attached to the aircraft fuselage through an aluminum alloy fitting. A maximum interbolt strain level, ϵ , of 3000 micro-inches per inch and an inter-bolt spacing of four hole diameters were taken as representative of such fittings.

Design conditions representative of an advanced fighter type aircraft were chosen. The joint was to carry a tensile load, N_x, of 20,000 lbf. per inch of chord. It was assumed that the joint fittings would be covered by an aerodynamic fairing, and a maximum joint length of ten inches was allowed.

To keep the joint manufacture as simple as possible, it was decided that the skin thickness would vary linearly in the joint and that for any given joint design, all holes

would be of the same diameter. To simplify the analysis it was decided to limit the candidate materials to balanced design laminates composed of zero and ± 45 degree plies of uniform thickness and material composition. No attempt was made to judge the effect of these restrictions on design efficiency.

The interface requirements determined by the aluminum fitting, and the geometric requirements to satisfy manufacturing simplicity, limited the possible joint designs and prevented full utilization of the high strength and high modulus properties available in the selected materials. It was felt, however, that even under these restrictions it would still be possible to demonstrate significant structural efficiencies by properly tailoring the laminates used in the wing skin.

C. RELIABILITY CONSIDERATIONS

Aerospace structural designs have had to meet difficult requirements for reliability. These designs have had to be sufficiently strong to carry the required loads and light enough to work in the aerospace environment. In addition to these requirements, critical components of aerospace structures have been expected to demonstrate that they are "fail-safe" and, in military applications at least, to some degree battle damage tolerant.

Early composite materials were judged inadequate for aerospace applications because they could not meet these reliability conditions. At first, quantity production of

advanced laminates was impossible because the required quality control technology did not exist. Wide batch-to-batch variation of the properties of these early materials justified only low confidence levels in their structural reliability. Although the early advanced composites were considered unreliable, materials research continued, driven by the anticipated structural efficiencies such materials could make possible. In 1973, Kaminski reported that NARMCO 5208/T300 graphite-epoxy laminates could meet the anticipated requirements for high strength, high modulus, and low weight and could be manufactured reliably and delivered with minimal batch-to-batch variation of material properties (Ref. 2). This was chosen as the structural material to be used in this study.

The requirement for battle damage tolerance and fail-safe design is met by a variety of methods in designs with conventional materials. These methods include built in high excess load bearing capacity, alternate load paths, and various crack stoppage stress relief devices. All of these techniques can be applied to design with composite materials. In addition to these techniques there is a "buffer strip" material fabrication technique applicable to laminated composites which provides an integral crack stoppage capability. Kaminski and Eisenmann explained this technique in Ref. 7.

In buffer strip design, integral "buffer strips" of low modulus, high fracture toughness material are included

in the laminate. These strips are spaced so that cracks originating in the high modulus load bearing primary strips are arrested when they run into the buffer strips. Using this technique, structures can be built in which cracks would be arrested before entire structural components failed. It appears that this could be an effective and efficient way to increase battle damage tolerance and the capability of a structural component to function after crack initiation.

D. RANGE OF THE STUDY

NARMCO 5208/T300 $\left[0/\pm 45\right]$ buffer strip and non-buffer strip joints were designed for widely varying laminate compositions, numbers and sizes of bolts. To simplify joint fabrication, the joint thickness was varied linearly between the inboard and outboard thicknesses, and all bolt holes for any given joint design were of the same diameter. To interface with the aluminum alloy fittings, the interbolt strain level was held to a maximum of 3000 micro-inches per inch, and the interbolt spacing was set at four hole diameters. The designs were compared to determine the effects of variation of hole size, number of holes, and laminate composition upon joint weight and excess bearing capacity.

Joint weight was considered a measure of the joint structural efficiency. Joint excess bearing capacity was considered a measure of allowable fabrication error. Since drilling holes in fibrous laminated composites has been

found to be both difficult and expensive, it was felt that the number of holes in any joint would be a measure of the relative joint fabrication cost (Ref. 5).

II. ANALYSIS OF THE NON-BUFFER STRIP JOINT

A. SIZING THE NON-BUFFER STRIP JOINT

Figure 1 is a schematic drawing showing the wing skin configuration used in the non-buffer strip joint design. All bolt holes are of the same diameter. For this analysis, wing taper is disregarded and the bolt hole centers are assumed placed in parallel rows and columns four hole diameters apart.

In the following theoretical development, it is assumed that the applied tensile load, N_x, and the applied shear load, N_{xy}, are constant across the outboard edge of the joint. Thus it is possible to size the joint considering only one column of bolts. It is further assumed that fittings were designed so that each bolt transfers the same portion, P, of the applied tensile load and that each bolt in the inboard row transfers, in addition to P, the same portion P_s of the shear load. In actual practice it is doubtful that this idealization could be achieved. However, it is a standard design assumption used in industry today (Ref. 8). This assumption implies a high resultant bolt load, R, in the inboard row of bolts.

From experimental analysis, it was known that layups of all ±45 degree laminae would have a superior bolt load

bearing capacity (Ref. 9). Since the wing skin would be required to carry no applied load beyond the inboard row of bolts, it was decided to take advantage of this high bearing load capacity for all designs by requiring that, at the inboard row of bolts, the skin be composed of 100 per cent ± 45 degree plies.

Considering a four-hole-diameters-wide column of bolts, the total tensile load L in the skin is given by

$$(1) \quad L = N_x 4D$$

where D is the bolt diameter used in the joint. If there are N bolts in each column,

$$(2) \quad P = \frac{L}{N} = \frac{N_x 4D}{N}$$

As explained in Ref. 10, the bearing stresses in a plate due to bolt loads are a function of the magnitude of the bolt load, the diameter of the bolt hole, and the effective thickness of the plate. For this study t^*, the effective bolt bearing thickness of the plate, is defined as follows:

for $t \leq 2D$, $t^* = t$

for $t > 2D$, $t^* = 2D$

where t is the plate thickness. This definition of effective bearing plate thickness is adopted to account for the fact that in thick plates loaded through a bolt hole the bolt loads tend to distribute themselves so that higher portions of the load are carried at the plate edges than at the plate center. The thickness definition was not chosen

through rigorous experimental or analytical processes but rather in light of engineering experience with metal plates. The bolt bearing stress, σ_{BR}, is

$$(3) \qquad \sigma_{BR} = \frac{\text{Bolt Load}}{D\ t^*}$$

On all but the inboard row of bolts,

$$(4) \qquad \sigma_{bx} = \frac{4\ N_x}{N\ \ t^*}$$

Since the inboard bolts in the non-buffer strip joints are assumed to react the shear as well as a share of the tensile load, they carry a bolt load, R, given by:

$$(5) \qquad R^2 = \left(\frac{N_x\ 4D}{N}\right)^2 + \left(N_{xy} 4D\ \right)^2$$

$$(6) \qquad R = 4DN_x \left[\left(\frac{1}{N}\right)^2 + \left(\frac{N_{xy}}{N_x}\right)^2 \right]^{\frac{1}{2}}$$

The bearing stress on these inboard bolts is

$$(7) \qquad \sigma_{BR} = \frac{4N_x}{t^*} \left[\left(\frac{1}{N}\right)^2 + \left(\frac{N_{xy}}{N_x}\right)^2 \right]^{\frac{1}{2}}$$

Test specimens composed of NARMCO 5208/T300 $[\pm 45]$ layers were found to be able to withstand

$$\sigma_{BR}\text{max} = 78000 \text{ lbf./in.}^2$$

when this load was applied through untorqued bolts (Ref. 9). This value is used in Eq. 7 to determine the minimum allowable joint thickness at the inboard bolt holes. In cases where this is a critical design parameter, the geometry illustrated in Fig. 2(a) is used to size the joint. Otherwise, the geometry illustrated in Fig. 2(b) is used.

The bolt hole center locations are numbered from one to N beginning with location 1 at the first or outboard bolt and ending with location N at the last or inboard bolt. To simplify the analysis, other locations in the joints are specified by a parameter η, defined relative to the bolt hole numbers. $\eta = 1$ means at the position of the first bolt hole center, $\eta = 2$ means at the position of the second bolt hole center, and $\eta = N$ means at the position of the inboard bolt hole center. Intermediate positions are defined as follows:

$\eta = 1.5$ means halfway between $\eta = 1$ and $\eta = 2$

$\eta = 2.5$ means halfway between $\eta = 2$ and $\eta = 3$

$\eta = N-.5$ means halfway between $\eta = N-1$ and $\eta = N$

The bypass force, F_{BP}, in the joint is defined as the tensile load passed from station η to station $\eta+1$, thus,

$$F_{BP} = N_x 4D \left(1-\frac{\eta}{N}\right) \qquad (\eta = \text{integer})$$

The bypass stress, σ_{tx}, is defined as

$$\sigma_{tx} = \frac{F_{BP}}{4Dt}$$

The bypass strain ϵ is defined as

$$\epsilon = \frac{\sigma_{tx}}{E_x}$$

where E_x is the tensile modulus of the material.

The strain level halfway between station η and $\eta+1$ is given by

$$(8) \quad \epsilon_{\eta+.5} = \frac{N_x \left(1 - \frac{\eta}{N}\right)}{t_{\eta+.5} \; E_{x_{\eta+.5}}} \qquad (\eta = \text{integer})$$

Both thickness and tensile modulus of the material are allowed to change with location within the joint.

The stress-strain behavior of a zero degree NARMCO 5208/T300 lamina is linear to failure. As reported in Ref. 2, the modulus of elasticity of such a lamina in the direction of the graphite fibers was determined by experiment to be 20.5 X 10^6 lbf./in.^{2}. The stress-strain behavior of a NARMCO 5208/T300 45 degree lamina is not linear. Tests performed in 1974, (Ref. 11), for a NARMCO 5208/T300 45 degree lamina under room temperature dry conditions reported the stress-strain behavior in the form of a secant modulus, which varied with strain level from 2.9 X 10^6 lbf./in.^{2} at zero strain to 1.8 X 10^6 lbf./in.^{2} at a strain level of 13000 micro-in./in. as shown in Fig. 3. (Secant modulus is the slope of a line through a point on the stress-strain curve and the origin.)

As explained in Ref. 3, the strain state for a balanced laminate composed of n layers is described by

$$(9) \quad \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} \\ A_{12} & A_{22} & A_{24} \\ A_{14} & A_{24} & A_{44} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix}$$

where $[N]$ is a vector of applied tensile and shear loads and $[\epsilon]$ is a vector describing the strain state of the plate. The components of the $[A]$ matrix are defined as

follows:

$$(10)\quad A_{ij} = \sum_{k=1}^{n} \bar{C}_{ij}^{k}\, t_k$$

where $\bar{C}_{ij}^{k}$ are the elements of the compliance matrix of the k^{th} lamina.

This study is concerned only with balanced design laminates made from laminae of uniform thickness and material composition and oriented at either zero or ± 45 degrees to the spanwise direction. For such laminates, it is seen from Eqs. 9 and 10 that the various moduli vary linearly with the per cent of zero degree plies in the laminate. Because the ± 45 degree data on secant modulus was available, and because the secant modulus is convenient for design use, a laminate value of secant modulus was calculated. Figure 4 shows the variation of secant modulus with laminate composition for NARMCO 5208/T300 $[0/\pm 45]$ material at ϵ = 3000 micro-in./in. It was prepared assuming that this modulus varies linearly between the experimentally determined values for such laminates with zero and 100 per cent zero degree plies. Figure 4 is used to determine the tension modulus of the various NARMCO 5208/T300 $[0/\pm 45]$ laminates used in the study joint designs.

The laminate composition at station η = 1.5 is assumed to be the same as that of the wing skin outboard of the joint. It determines the tensile modulus at this position. Equation 8 is then used to determine $t_{1.5}$.

The laminate composition at station η = N-.5 is initially

determined by assuming that the percentage of zero degree plies varies linearly from station $\eta = 1.5$ to station $\eta = N$ where the laminate is to be composed of 100 per cent ± 45 degree plies. An additional constraint, applicable only to the non-buffer strip joint, is that the laminate at station $\eta = N-.5$ can have no fewer than 5 per cent zero degree plies. This is done to ensure that there are sufficient load bearing zero degree plies to carry the bypass load between the second to last and the last, or inboard, row of bolts.

Having fixed the laminate composition, the tensile modulus and, through Eq. 8, the thickness at station N-.5 are determined.

The remaining joint thicknesses are determined assuming a linear variation of joint thickness between $\eta = 1.5$ and $\eta = N-.5$. Laminate compositions midway between each pair of adjacent bolts are determined using the thickness distribution and the desired strain level.

$$(11) \quad E_{x_{\eta+.5}} = \frac{N_x \left(1 - \frac{\eta}{N}\right)}{t_{\eta+.5}\,\epsilon} \qquad (\eta = \text{integer})$$

The tension modulus, E_x, of the material is assumed to vary linearly with distance between the values determined from Eq. 11. Thus,

$$(12) \quad E_{x_\eta} = \frac{E_{x_{\eta-.5}} + E_{x_{\eta+.5}}}{2} \qquad (\eta = \text{integer})$$

These modulus values determine the laminate composition throughout the joint.

Joints are sized using 0.25, 0.375, 0.4375, and 0.5 inch bolts with laminate compositions at the first bolt varying from 60 to 10 per cent zero degree plies. These laminate compositions cover the range over which the Eisenmann strength model to be described in the next section was considered accurate. In all cases, the same interbolt strain level, 3000 micro-inches per inch, is maintained. The minimum number of bolts used in any joint design is three. This is done to provide a mechanism by which the desired strain level can be maintained. The maximum number of bolts in any joint is determined by the desired interbolt spacing of four bolt hole diameters and the requirement that the joint length not exceed ten inches.

B. DETERMINING THE STRENGTH OF A NON-BUFFER STRIP JOINT

Waddoups, Eisenmann, and Kaminski, in Ref. 12, showed experimentally that graphite-epoxy laminates are statically brittle and exhibit many of the failure characteristics of brittle materials first explained by Griffith in Ref. 13. They formulated a model which assumed that crack growth behavior in graphite-epoxy laminates was a function of stress intensity and critical energy level, and they verified their model by experiment.

In Ref. 14, Eisenmann continued this work and developed a bolted joint strength model for composite materials. This model accounted for the material ultimate strengths, local stress intensity factors, and geometric width correction factors. With the Eisenmann model, it was

possible to calculate the total stress at any point on a loaded circular hole in an orthotropic plate by linearly combining the various stresses acting upon the plate using Eq. 13. Because of the internal curve fitting techniques used in this model, it was considered to give accurate results for laminate compositions varying from 10 to 60 per cent zero degree plies (Ref. 14).

$$(13)\quad \sigma^{i} = \lambda^{i}_{tx} f(\tfrac{a}{r})^{i}_{tx} \sigma_{tx} + \lambda^{i}_{ty} f(\tfrac{a}{r})^{i}_{ty} \sigma_{ty} + \lambda^{i}_{xy} f(\tfrac{a}{r})^{i}_{xy} \sigma_{xy} + \lambda^{i}_{bx} f(\tfrac{a}{r})^{i}_{bx} \sigma_{bx} + \lambda^{i}_{by} f(\tfrac{a}{r})^{i}_{by} \sigma_{by}$$

where:

i = indicator of the exact position on the hole.

λ^{i}_{α} = finite width correction factor at location i for applied load condition α

$f(\frac{a}{r})^{i}_{\alpha}$ = effective isotropic stress concentration factor at location i for applied load condition α

σ_{α} = applied stress for load condition α

σ^{i} = net tangential stress at location i

α = subscript denoting applied load condition as follows:

xy = shear

tx = tension in x - direction

ty = tension in y - direction

bx = bearing in x - direction

by = bearing in y - direction

The stress definitions are sketched in Fig. 5. The λ and f factors are determined by plate geometry and material composition.

The failure modes of NARMCO 5208/T300 $[0/\pm45]$ plates with loaded holes were determined by test (Ref. 14). Crack initiation in the test specimens most often occurred on the hole edge at positions β=0, $\pm$45, $\pm$90, $\pm$135, or 180 degrees measured from the X axis. From symmetry it was determined that all these failure modes could be adequately described by description of the failure modes encountered at β =0, 45, and 90 degrees. Laminate strength for each of these positions was determined by experiment (Ref. 14). In these tests the bolts used to load the holes were untorqued.

The Eisenmann static strength model was used to calculate stress intensity and geometric width correction factors based upon an interbolt spacing of four hole diameters. Equation 13 was used to prepare Figs. 6-29 which define the failure modes expected for laminates whose composition varies from 60 to 10 per cent zero degree plies with 0.25, 0.375, 0.4375, and 0.50 inch holes. Only the effects of σ_{xy}, σ_{tx}, and σ_{bx} were considered in the preparation of these curves.

One of the parameters of interest in this study is the excess bearing capacity of each joint design. For purposes of this study the excess bearing capacity of the joint is defined as the smallest excess bearing capacity at any

bolt hole in the joint. At each hole a representative stress load, S_1, which accounts for the combined bearing and bypass stresses is calculated as follows:

$$(14) \quad S_1 = \left[\sigma^2_{bx} + \sigma^2_{tx}\right]^{\frac{1}{2}} \quad \text{as loaded}$$

A representative ultimate strength, S_2, is calculated from:

$$(15) \quad S_2 = \left[\sigma^2_{bx} + \sigma^2_{tx}\right]^{\frac{1}{2}} \quad \text{at failure}$$

where the failure state is the state at which

$$(16) \quad \left[\frac{\sigma_{bx}}{\sigma_{tx}}\right]_{\text{failure}} = \left[\frac{\sigma_{bx}}{\sigma_{tx}}\right]_{\text{loaded}}$$

Then, as shown in Fig. 30, the excess bearing capacity, B is defined as

$$(17) \quad B = \frac{S_2 - S_1}{S_1}$$

$B > 0$ at all holes in a joint implies that there is some margin of safety.

$B = 0$ at any hole in a joint implies that there is no margin of safety.

$B < 0$ at any hole in a joint implies that the joint would fail under the applied load.

The effect of laminate composition and bolt hole size on excess bearing capacity is shown in Figs. 31, 32, and 33.

C. DETERMINING THE WEIGHT OF A NON-BUFFER STRIP JOINT

The weights per inch of chord for the non-buffer strip joints were calculated by multiplying the cross-sectional

areas of each joint by the density of NARMCO 5208/T300 graphite-epoxy laminate material. The effect of laminate composition, hole size, and number of bolt holes upon joint weight is shown in Figs. 34, 35, and 36.

III. ANALYSIS OF THE BUFFER STRIP JOINT

A. SIZING THE BUFFER STRIP JOINT

Figure 37 is a schematic of the buffer strip joint design. All bolt holes are of the same diameter. Except for the inboard row, all bolt holes are placed in the buffer strips. This placement was chosen for two reasons:

1. It took advantage of the high bearing capacity of NARMCO 5208/T300 $[\pm 45]$ laminates.
2. It reduced stress concentrations in the heavily loaded primary strips.

The bolt holes in the buffer strips are spaced so that there is a distance of four hole diameters between adjacent hole centers. Two bolt holes are placed in each primary strip which is located between two buffer strips. These bolt holes are placed in a row with the inboard bolt in the buffer strip. In the joint analysis, wing taper is disregarded and the spanwise edges of the buffer and primary strips are considered parallel. It is assumed that joint thicknesses at any position are the same in the buffer and primary strips.

The buffer strips used in the joints analyzed in this study are four hole diameters wide, and the two primary

strips are each 3.335 hole diameters wide. This gives an overall buffer strip joint width of 10.67 hole diameters. These dimensions were chosen because it was felt that they were representative of a geometry which could be used in an advanced fighter type aircraft wing skin application. No attempt is made to justify these dimensions either analytically or experimentally, and no attempt is made to assess the effect of this choice of dimensions upon joint efficiency.

In the following theoretical development, as in the case of the non-buffer strip joint, a tensile load N_x and a shear load N_{xy} are assumed constant across the outboard edge of the joint. It is also assumed that fittings were designed so that each row of bolts in the joint transfers the same portion P of the applied tensile load and so that the shear load is reacted by the inboard row of bolts, each of the inboard bolts carrying an equal share of the shear load as well as a share of the tensile load. To utilize the high bearing capacity of a NARMCO 5208/T300 $[\pm 45]$ laminate, it was decided to impose a requirement that at the inboard row of bolts, the primary strips be composed of 100 per cent ± 45 degree plies.

Considering the joint sketched in Fig. 37, the load, L, on a single buffer strip joint, is given by:

$$(18) \quad L = N_x AD$$

where AD is the overall joint width. The bolt load in all bolts except those in the inboard row is given by

$$(19) \quad P = \frac{L}{M} = \frac{N_x AD}{M}$$

where M is the number of rows of bolts in the joint.

Since the shear load N_{xy} and a total tensile bolt load P are reacted equally by each of the three bolts in the inboard row of bolts, the resultant load, R, on each of these bolts is calculated from

$$(20) \quad R^2 = \left[\frac{N_x AD}{3(M)}\right]^2 + \left[\frac{N_{xy} AD}{3}\right]^2$$

$$(21) \quad R = \frac{ADN_x}{3}\left[\left[\frac{1}{M}\right]^2 + \left[\frac{N_{xy}}{N_x}\right]^2\right]^{\frac{1}{2}}$$

As in the non-buffer strip joint design, the desired interbolt strain level, 3000 micro-inches per inch, is the primary consideration determining the joint geometry for the buffer strip design. Locations in the joint are described by station numbers, just as in the non-buffer strip design. In the case of the buffer strip design, however, the station numbers vary from η=1, which corresponds to the location of the outboard bolt-hole center, to station η=M which corresponds to the location of the row of centers of the inboard bolts.

The thickness at station η=1.5 is determined by the applied load, the joint geometry, the joint material composition, and the desired interbolt strain level. The average modulus of the joint, $E_{x_{ave}}$, as defined in Ref. 7, is used in calculating this thickness.

$$(22) \quad E_{x_{ave}} = \frac{D(A-W_B)\ E_{x_1} + W_B D E_{x_2}}{AD}$$

where $W_B D$ = width of the buffer strip

E_{x_1} = tensile modulus of the primary strip

E_{x_2} = tensile modulus of the buffer strip

As shown in Fig. 4, E_{x_1} is determined by the percentage of zero degree plies in the primary laminate. As shown in Fig. 3, E_{x_2} varies with strain level.

The thickness at station $\eta = 1.5$ is derived from

$$(23) \quad \epsilon = \frac{ADN_x \left[1-\frac{1}{M}\right]}{ADt\ E_{x_{ave}}}$$

where ϵ is the desired interbolt strain level. Thus,

$$(24) \quad t_{1.5} = \frac{AN_x \left[1-\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right] \epsilon}$$

Similarly,

$$(25) \quad t_{M-.5} = \frac{AN_x \left[\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right] \epsilon}$$

The bolt bearing stress in the inboard row of bolts is given by

$$(26) \quad \sigma_{BR} = \frac{R}{t^*}$$

where t^*, the effective bearing thickness, is defined as in the non-buffer strip joint.

As stated earlier, the bearing strength of NARMCO 5208/T300 $[\pm 45]$ degree laminates is 78000 lbf./in.^{2}. This determines the minimum allowable joint thickness at station

$\eta = M$ for any given load condition.

The laminate composition in the primary strips is initially assumed to vary linearly with distance from the composition at station $\eta = 1.5$, where it is the same as that of the plate outboard of the joint, to 100 per cent ± 45 degree plies at station $\eta = M$ where the applied shear loads are reacted.

The joint thickness at station $\eta = M-.5$ is determined by the desired interbolt strain level.

$$(27) \quad t_{M-.5} = \frac{AN_x}{M\left[(A-W_B)\ E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

The remaining joint thicknesses are then determined geometrically from those at $\eta = 1.5$ and $\eta = M-.5$ using the same techniques as for the non-buffer strip joint. If possible, a cross-section similar to that shown in Fig. 2(b) is used. When this yields a thickness at the last bolt which is too small, using the maximum bearing stress criteria discussed above and Eq. 26, a cross-section similar to that of Fig. 2(a) is used.

Having fixed the joint geometry, the laminate composition is determined by the requirement that the design strain level be maintained between each pair of bolt holes.

At any station $\eta = k+.5$, k=1, 2, ..., M-1,

$$(28) \quad \epsilon = \frac{N_x A\left[1-\frac{k}{M}\right]}{\left[E_{x_1}(A-W_B) + E_{x_2} W_B\right]t}$$

Since the composition of the buffer strips is fixed and E_{x_2}

is determined by the interbolt strain level, it is necessary to vary E_{x_1} and hence the composition of the primary strips to maintain the desired strain level as the bypass loads in the joint vary from hole to hole.

$$(29) \quad E_{x_1} = \frac{1}{(A-W_B)} \left[\frac{N_x A\left[1-\frac{k}{M}\right]}{t_\eta \; \epsilon} - W_B E_{x_2} \right]$$

It is assumed that E_{x_1} varies linearly with distance between stations $\eta = k-.5$ and $\eta = k+.5$. Thus

$$(30) \quad E_{x_{1_k}} = \frac{E_{x_{1_{k+.5}}} + E_{x_{1_{k-.5}}}}{2}$$

Determination of $E_{x_{1_k}}$ determines the required percentage of zero degree plies in the primary laminate at station k.

Having determined the joint geometry and laminate composition, it is then possible to determine the joint weight and excess bearing capacity.

B. DETERMINING THE STRENGTH OF A BUFFER STRIP JOINT

The bolt load P is not carried entirely in the buffer strip. Part of it is transmitted, in shear, to the primary strips. This load splitting is shown in Fig. 38 in which P_1 is the portion of the bolt load P reacted through each of the primary strips and P_2 is the portion of P reacted through the buffer strip. The relationship between P_1 and P_2 is determined analytically as follows:

Referring to Fig. 38,

$$(31) \quad P = 2P_1 + P_2$$

The bypass strain levels in the buffer and primary strips

are assumed the same. Defining

$$(32)\quad 2W_1D = AD - W_BD$$

$$(33)\quad 2W_1 = (A - W_B)$$

$$(34)\quad \frac{P_1}{E_{x_1}W_1t} = \frac{P}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

Rearranging Eq. 34 gives an expression for P_1 in terms of P, the joint composition, and the joint geometry.

$$(35)\quad \frac{P_1}{P} = \frac{E_{x_1}W_1t}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

This equation is rewritten in the form

$$(36)\quad P_1 = A_nP$$

where A_n is determined by knowledge of the joint laminate composition, geometry, and strain level.

$$(37)\quad A_n = \frac{E_{x_1}W_1}{2E_{x_1}W_1 + E_{x_2}W_B}$$

The bolt load splitting discussed above is dependent upon the ability of the ± 45 degree laminae to transfer a shear load, F_S, from the buffer to the primary strips. It was experimentally determined that failure of this load transferring mechanism could be expected when the shear stress in these fibers, f_s, reached a magnitude of 90,000 lbf./in.^{2} (Ref. 9). At any station in the joint

$$(38)\quad F_{S_\eta} = \frac{P_1}{4Dt_{\pm 45}} \qquad (\eta = \text{integer})$$

where $t_{\pm 45}$ is the total thickness of ± 45 degree laminae through which P_1 is transferred. Defining Z = percentage of zero degree plies in the primary strips,

$$(39) \quad t_{\pm 45} = t \frac{(100-Z)}{100}$$

$$(40) \quad f_{s_\eta} = \frac{N_x \; A \; A_\eta}{4M \; t_\eta \frac{(100-Z)}{100}} \qquad (\eta = \text{integer})$$

The shear stress f_s varies from bolt to bolt in a given buffer strip joint design. In all designs, however, the highest values for f_s occur at the first bolt hole. Thus f_{s_1} determines the upper limit on the percentage of zero degree plies in the primary strip laminate. From Eq. 24,

$$(41) \quad t_1 = \frac{AN_x \left[1-\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right]\epsilon}$$

Fig. 4 yields the following relationship between laminate composition and modulus,

$$(42) \quad E_{x_1} = 10^6 \left[20.7-(100-Z) \left[\frac{20.7-2.8}{100}\right]\right] (\text{lbf./in.}^2)$$

Then,

$$(43) \quad \frac{100-Z}{100} = \left[20.7 - \frac{E_{x_1}}{10^6}\right] \frac{1}{17.9}$$

Substituting Eqs. 37, 41, and 43 into Eq. 40,

$$(44) \quad f_{s_1} = \frac{W_1 E_{x_1}}{(M-1)} \frac{1}{4} \frac{\epsilon_1}{\left[20.7 - \frac{E_{x_1}}{10^6}\right]} \frac{1}{17.9}$$

From Eq. 44 it is seen that f_{s_1} is determined by the joint

geometry, laminate composition, and design strain level. Fixing the joint geometry and the design strain level determines the maximum allowable modulus for the primary strips and hence provides an upper limit on the per cent zero degree plies which can be used in the primary strips. This limit is determined by setting f_{s_1} equal to its experimentally determined maximum, 90,000 lbf./in.^{2} and using Eq. 45 to determine $E_{x_{1_{max}}}$.

(45)

$$E_{x_{1_{max}}} = \frac{4(M-1)(20.7)f_{s_1}}{17.9(W_1)(\epsilon_1)10^6 + 4(M-1)f_{s_1}} \quad 10^6(\text{lbf./in.}^2)$$

Under the design conditions applicable to this study, Eq. 45 implies an upper limit of 92 to 98 per cent zero degree plies in the primary strips.

Three other failure modes of a buffer strip joint were found to be most probable under N_x and N_{xy} loading (Ref. 15). Type I and Type II failure occurred in the buffer strip at the loaded bolt holes. Type I failure was characterized by radial cracks at 45 degrees to the X axis. Type II failure was characterized by radial cracks originating at the edge of the hole at 90 degrees to the X axis. Type III failure occurred when the load bearing fibers in the primary strips were broken. These three failure modes are sketched in Fig. 39.

The Type I failure mode characterizes the interaction of bearing and shear stresses in the buffer strip. It was found to be essentially independent of the bypass

stress. The load curves describing this failure mode were derived from experimental results (Ref. 15). Single hole specimens of buffer strip joint material were clamped in test machines along either one or two sides as shown in Fig. 40. With the bolts torqued, the specimens were loaded, and the failure bolt load stresses measured. Tests were run for specimens with 0.25 inch and 0.4375 inch diameter holes. For the doubly clamped test cases Type I failure occurred under the following loads:

$$D = 0.250 \text{ in. } \sigma_{bx_{max}} = 151{,}200 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx_{max}} = 144{,}100 \text{ lbf./in.}^2$$

For the test specimens clamped at only one edge, Type I failure occurred under the following loads:

$$D = 0.25 \text{ in. } \sigma_{bx} = 111{,}000 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx} = 107{,}000 \text{ lbf./in.}^2$$

Satisfactory test results for the pure shear load case could not be obtained.

An attempt was made to approximate the shear effects by superposition of the singly and doubly clamped test results. The finite element computer program ISANIS, listed in Appendix A, was used to analyze the stress concentration field in various orthotropic plates. It was found that for a square plate made from uniform material with a central hole and sides at least four hole diameters in length, the stress field at the hole due to pure applied

shear could be closely approximated by an appropriate superposition of the stress fields resulting from singly and doubly clamped load cases. The superposition used is shown schematically in Fig. 41. The key assumption in this superposition is that the moment reaction which is representative of the single clamped edge load case can be replaced by a couple of equal magnitude resulting from shear loads applied on the upper and lower edges of the specimen. This assumption is really an application of the St. Venant principle that points in a body removed from load application locations react to the load applied rather than its mechanism of application. ISANIS was used to test this assumption and, under the conditions stated above, it was found to be reasonable.

The test specimens were thin and hence $t^*=t$. Therefore,

$$(46) \quad P = \sigma_{bx} D t$$

For the double clamped specimen, the reaction force, F_{R_D}, is given by

$$(47) \quad F_{R_D} = \frac{P}{2} = \frac{\sigma_{bx} D t}{2}$$

Assuming that this force is uniformly distributed across the clamped edges, an edge stress is defined

$$(48) \quad \sigma_{R_D} = \frac{F_{R_D}}{\ell D t}$$

where ℓ X D equals the length of the clamped edge.

For the single clamped specimen, the reaction force,

F_{R_S}, is equal to the applied bolt load P. The reaction moment, m, is given by

$$(49) \quad m = P\frac{\ell D}{2}$$

This moment is approximated by a couple of the same magnitude formed by forces acting on the edges of the specimen which are normal to the clamped edge. For square specimens of side length ℓD, the magnitude of these forces, F, is given by

$$(50) \quad F\ell D = m$$

$$(51) \quad F\ell D = \frac{P\ell D}{2}$$

$$(52) \quad F = \frac{P}{2}$$

This development is also shown schematically in Fig. 41.

Assuming that each force F is distributed uniformly along the specimen edge upon which it is applied, an edge shear is defined

$$(53) \quad \sigma_{xy} = \frac{F}{\ell Dt}$$

$$(54) \quad \sigma_{xy} = \frac{P}{2\ell Dt}$$

For the test specimens each side was four hole diameters in length. From Eqs. 47 and 54, the superposition yields

$$(55) \quad \sigma_{xy} = \frac{\sigma_{bx}}{8}$$

Under these assumptions, the experimental data listed previously yield the following failure states:

$$D = 0.25 \text{ in.}$$

$\sigma_{xy} = 0$ $\qquad$ $\sigma_{bx} = 151{,}200$ lbf./in.^{2}

$\sigma_{xy} = 13{,}875$ lbf./in.^{2} $\qquad$ $\sigma_{bx} = 111{,}000$ lbf./in.^{2}

$$D = 0.4375 \text{ in.}$$

$\sigma_{xy} = 0$ $\qquad$ $\sigma_{bx} = 144{,}100$ lbf./in.^{2}

$\sigma_{xy} = 13{,}375$ lbf./in.^{2} $\qquad$ $\sigma_{bx} = 107{,}000$ lbf./in.^{2}

Figure 42 was prepared from these data points assuming that the ultimate bearing stress-shearing stress interaction curve was linear for the graphite-epoxy laminates used in this study.

Buffer strip Type II failure curves were experimentally determined for buffer strip joint specimens with 0.4375 inch bolt holes, a buffer strip width of 1.5 inches, and primary strips each 1.25 inches wide (Ref. 15). The hole centers were spaced four diameters apart. These tests were run with primary strips composed of 30 per cent zero degree plies and 50 per cent zero degree plies with bolt loads applied through torqued bolts. These test specimens were thin enough so that $t^*=t$. Tests were also run on buffer strips alone to determine the ultimate bypass stress of the buffer strip material (Ref. 15). With no applied bearing stress it was found that the ultimate bypass stress in the buffer strip material was 25000 lbf./in.^{2}. The results of this series of tests indicated that the Type II failure mode for these test specimens could be closely described by the empirical relationship

$$(56) \quad \sigma_{net\ buffer} = \frac{P_2 + .25(2P_1)}{D(W_B - 1)\ t_2} + \sigma_{tx_{2_{net\ area}}}$$

$\sigma_{net\ buffer}$ is the ultimate bypass stress in the buffer strip normalized to the width of the buffer strip minus the diameter of the bolt hole. $\sigma_{tx_{2_{net\ area}}}$ is the actual bypass stress in the buffer strip normalized to the width of the buffer strip minus the hole diameter. Thus,

$$(57) \quad \sigma_{net\ buffer_{max}} = 25000\ \text{lbf./in.}^2 \left(\frac{W_B}{W_B - 1}\right)$$

$$(58) \quad \sigma_{tx_{2_{net\ area}}} = \frac{\sigma_{tx_2}\ W_B}{(W_B - 1)}$$

Assuming that the strain levels in the buffer and primary strips are the same at any station η,

$$(59) \quad \frac{\sigma_{tx_1}}{E_{x_1}} = \frac{\sigma_{tx_2}}{E_{x_2}} = \epsilon$$

Thus,

$$(60) \quad \sigma_{tx_{2_{net\ area}}} = \frac{\sigma_{tx_1}}{E_{x_1}} E_{x_2} \frac{W_B}{(W_B - 1)}$$

$$(61) \quad \sigma_{bx} = \frac{P}{Dt}$$

(62)

$$25000 \left(\frac{W_B}{W_B - 1}\right) = \frac{\sigma_{bx}(1 - 1.5\ A_n)}{(W_B - 1)} + \frac{\sigma_{tx_1}\ E_{x_2}}{E_{x_1}} \frac{W_B}{(W_B - 1)}$$

For a fixed laminate composition in the primary strips, different values of σ_{tx_1} produce different strain levels. This implies variation in E_2, the modulus of the buffer

strip material, with bypass stress in the primary strips. This variation in modulus explains why the Type II failure is not linear in σ_{bx} and σ_{tx_1}.

Type III failure is characterized by fracture of the zero degree fibers in the primary strips. As reported in Ref. 2, this failure mode is encountered when the strain level in these strips reaches 10,000 micro-inches per inch. This failure mode is mathematically predicted by considering the presence of both the bypass stress and the stress due to P_1 in the primary strips. Thus,

$$(63)\quad \sigma_{tx_1} + \frac{A_n P}{W_1 D t} = \sigma_1$$

$$(64)\quad \sigma_{1_{ult}} = \epsilon_{ult} E_1$$

$$(65)\quad P = \sigma_{bx} Dt^*$$

$$(66)\quad \sigma_{tx_1} + \frac{A_n \sigma_{bx} Dt^*}{(A-W_B)\ Dt} = \epsilon_{ult} E_1$$

The results of these tests are summarized in Fig. 43. The Type I failure line is drawn for a zero shear case and is determined from experiment with a double clamped test specimen, the Type II failure lines are drawn from Eq. 62, and the Type III failure lines are drawn from Eq. 66. For the joints used in this study, Eq. 62 becomes

$$(67)\quad \frac{\sigma_{bx}(1-1.5A_n)}{3} + \sigma_{tx_1} \quad \frac{E_{x_2}}{E_{x_1}} \frac{4}{3} = 25\ \frac{4}{3}$$

and Eq. 66 becomes

$$\sigma_{tx_1} + \frac{A_n \sigma_{bx}}{6.67} = \epsilon_{ult} E_1 \tag{63}$$

Figures 44 and 45 describe the expected failure states for the buffer strips used in the study joints. The Type I failure lines on these figures are taken from the zero shear ultimate bearing stresses indicated on Fig. 42. The Type II and Type III failure lines shown in these figures are drawn from Eqs. 67 and 68.

Excess bearing capacity calculations were made for buffer strip joints just as had been done for non-buffer strip joints. In the case of the buffer strip joints, however, only 0.25 and 0.4375 inch holes were considered since Type I failure test data was available only for these hole sizes. The results of these calculations are presented in Figs. 46 and 47.

C. DETERMINING THE WEIGHT OF A BUFFER STRIP JOINT

The weight per inch of chord of each buffer strip design was calculated just as had been done for the non-buffer strip joints by multiplying the cross-sectional area of each joint by the density of the NARMCO 5208/T300 graphite-epoxy material used in the study. The variation of joint weight with primary strip laminate composition, bolt hole size, and number of bolts is shown in Figs. 48 and 49.

IV. DISCUSSION OF RESULTS

A. NON-BUFFER STRIP JOINTS

The following generalizations about non-buffer strip joints were found to be valid:

1. For a given hole size, the joint weight decreased as the percentage of zero-degree plies in the joint increased.
2. For a given hole size, the fewer the number of bolt holes per unit chord, the lighter the joint.
3. The smaller the bolt holes, the lighter the joint could be made.
4. The smaller the bolt holes, the greater the range in number of allowable bolt holes per unit chord.
5. For a given laminate composition, the smaller the bolt holes the larger the minimum number of bolt holes per unit chord required.
6. The smaller the bolt holes, the larger the allowable range of laminate composition.
7. For a given laminate composition, the smaller the bolt holes, the larger the excess bearing capacity.

B. BUFFER STRIP JOINTS

Observations 1-6, above, are also true for buffer strip joints. The excess bearing capacity of the buffer

strip joint, as seen in Figs. 48 and 49, is, in a gross sense, independent of hole size and more heavily influenced by laminate composition, the number of holes, and the load condition.

C. COMPARISON OF NON-BUFFER STRIP AND BUFFER STRIP JOINTS

In Ref. 16 it is explained that the bearing strength of bolted plates is higher when the bearing loads are applied through torqued bolts than when they are applied through untorqued bolts. Since the failure modes of the buffer strip joints were derived from experiments in which the bolt loads were applied through torqued bolts, and since the failure modes of the non-buffer strip joints were derived from experiments in which the bearing loads were applied through untorqued bolts, this may explain at least part of the apparently higher excess bearing capacities available with buffer strip joints. Buffer strip joints generally weigh more than non-buffer strip joints. The fact that buffer strip joints require fewer bolts per inch of chord than non-buffer strip joints can be used to offset some of this weight difference if bolt weights are included in the net joint weight. The reduced number of bolts per inch of chord possible with buffer strip joints should also reduce joint fabrication costs by reducing the number of drilling operations required. Buffer strip joints can be constructed for a larger range of laminate composition than non-buffer strip joints.

V. CONCLUSIONS AND RECOMMENDATIONS

The design methodology used in this study is shown to be capable of producing workable joint designs. The wide range of weights and excess bearing capacities exhibited by the different joint designs indicates that the joint efficiencies achieved by geometric substitution of advanced composites for conventional structural materials were probably minimal when compared with those which would be achieved by designs which took advantage of the special high strength and high modulus properties of advanced composites.

Under the design conditions adopted in this study, buffer strip joints were found to be stronger and more cheaply manufactured than non-buffer strip joints. These advantages are offset by the increased joint weights characteristic of buffer strip joints. In spite of this weight penalty, it is felt that the high excess bearing capacity and integral crack stoppage capability of buffer strip joints makes them promising candidates for aerospace applications.

It is felt that Figs. 31-36 and 46-49 can be used to compare various design proposals and to estimate the costs of variation in laminate composition, hole size, or number of holes. No attempt was made to determine the effect of the design limitations summarized in TABLE I which were placed upon allowable joint geometry and

composition by such factors as manufacturing considerations and fitting interface requirements. It is recommended that the effect of these restrictions be determined by an analysis similar to this one with the restrictions removed.

It is recommended that application of the buffer strip technique to critical components be preceded by further experimentation to more accurately determine the behavior of buffer strip joints under shear loads.

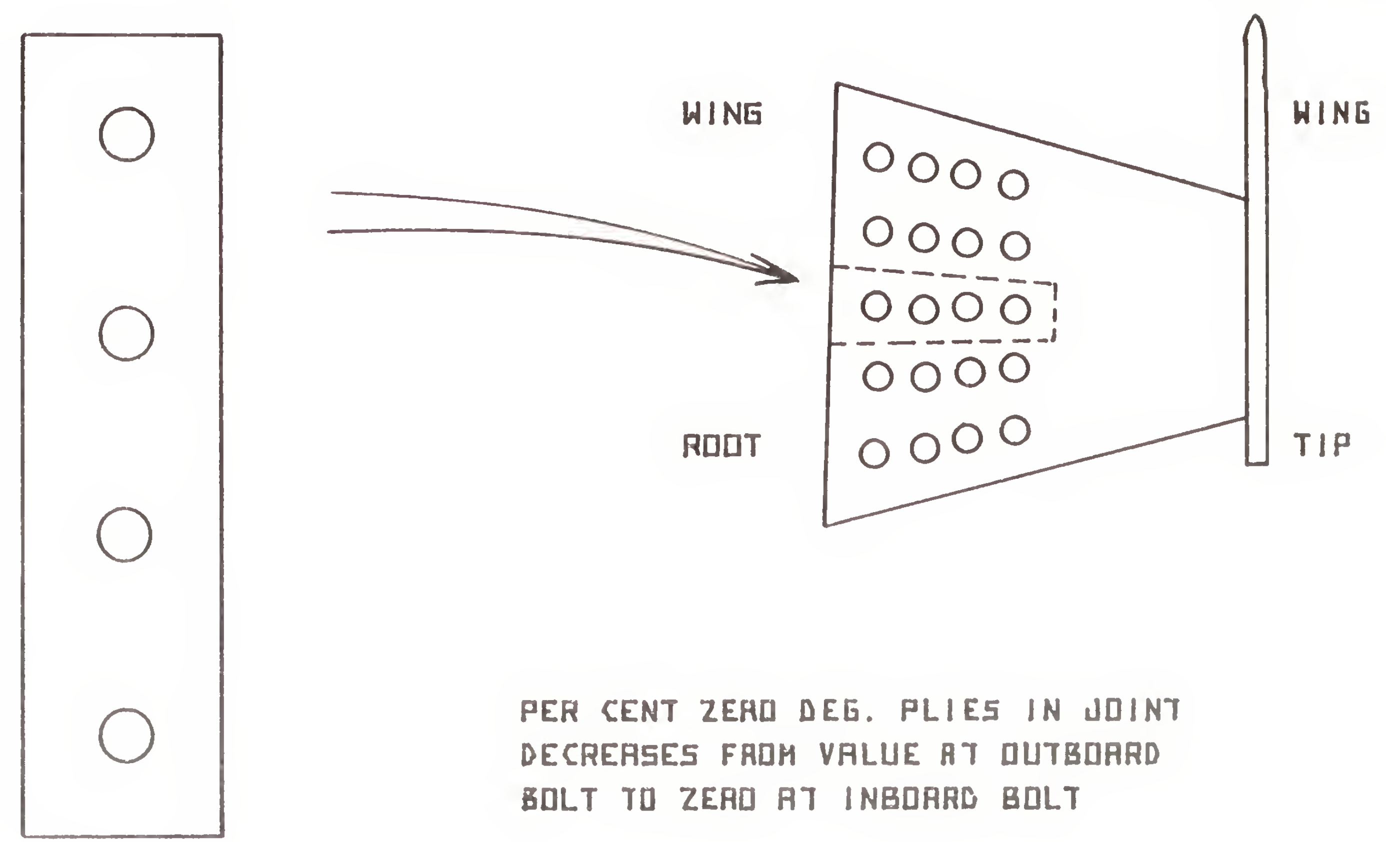
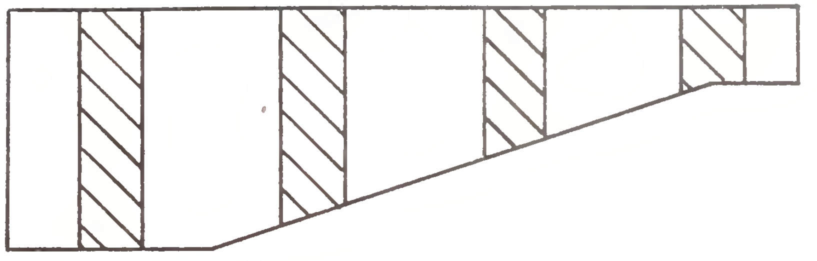


FIGURE 1. SCHEMATIC OF A WING WITH A NON-BUFFER STRIP JOINT



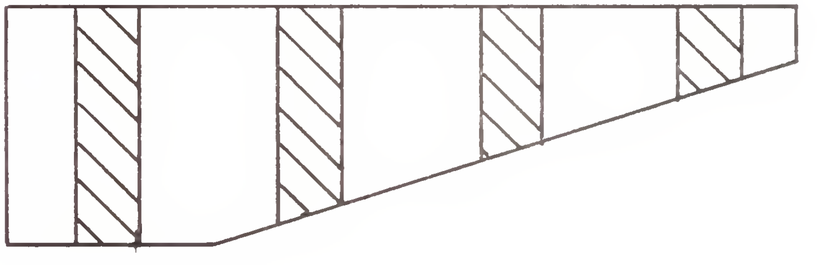


FIGURE 2. PERMISSIBLE JOINT CROSS SECTIONS

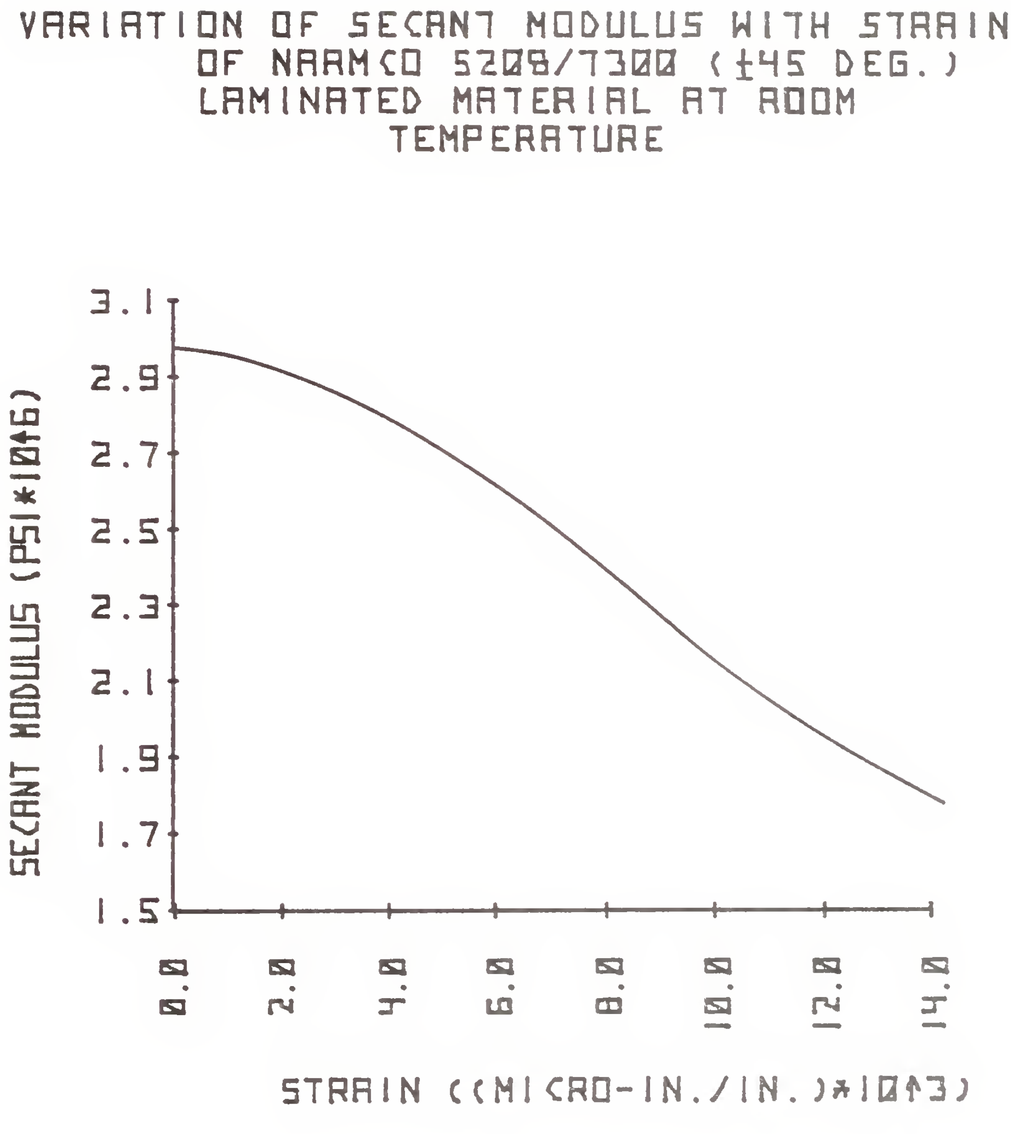


FIGURE 3. VARIATION OF SECANT MODULUS WITH STRAIN OF NARMCO 5208/T300 (±45 DEG.) LAMINATED MATERIAL AT ROOM TEMPERATURE

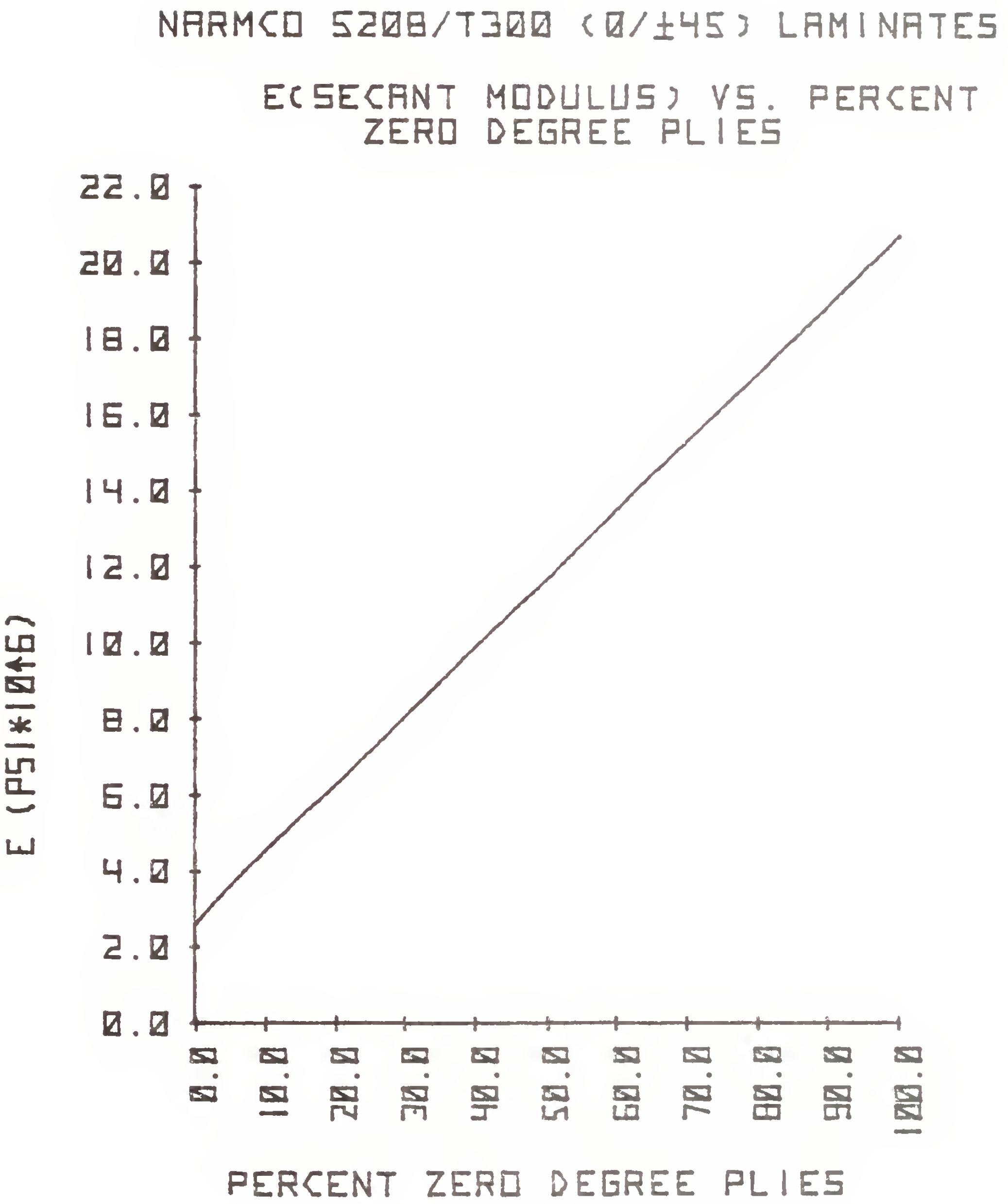


FIGURE 4. SECANT MODULUS OF NARMCO 5208/T300 [0/±45] LAMINATES VS. PER CENT ZERO DEGREE PLIES

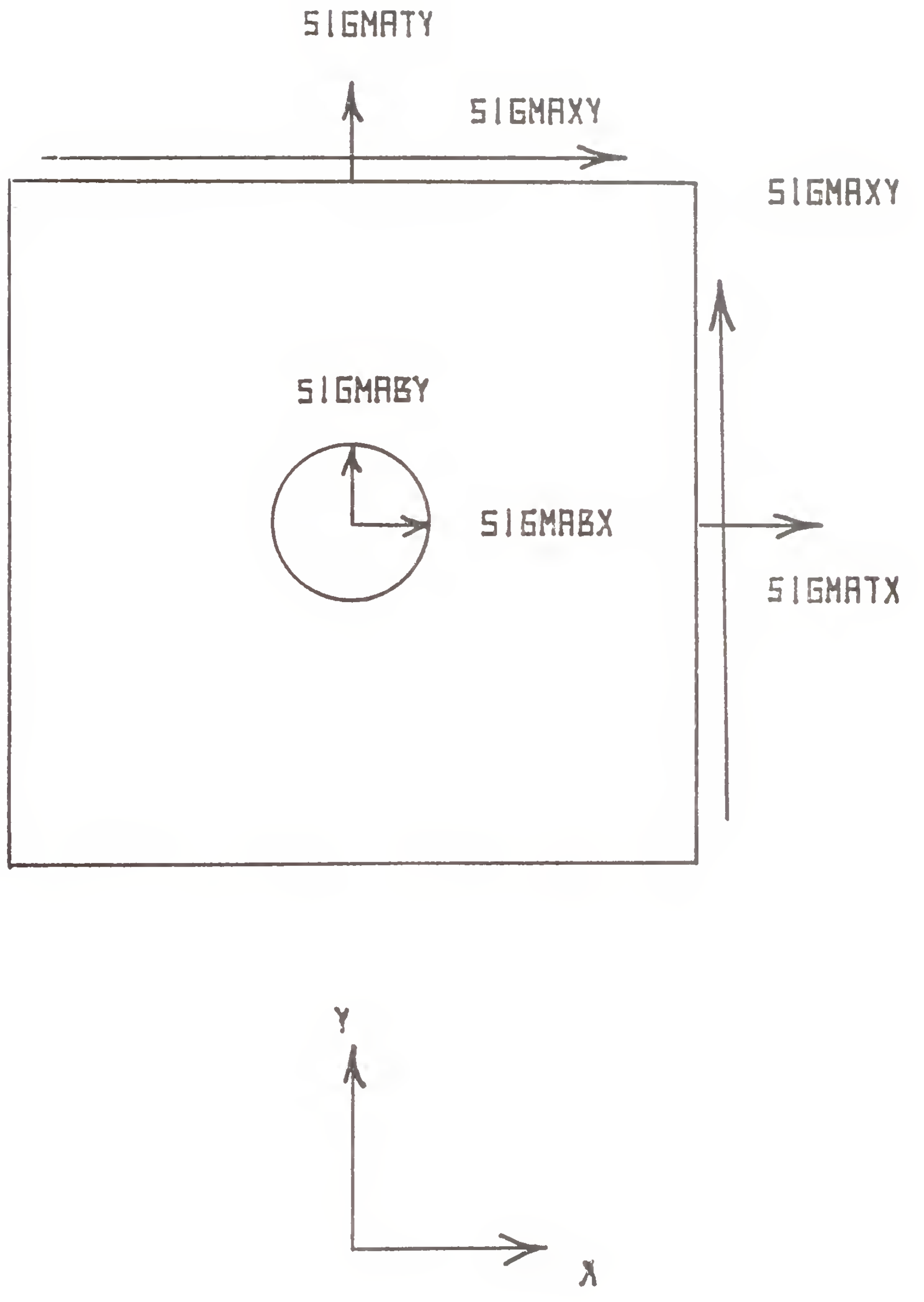


FIGURE 5. BOLTED JOINT APPLIED STRESS DEFINITIONS

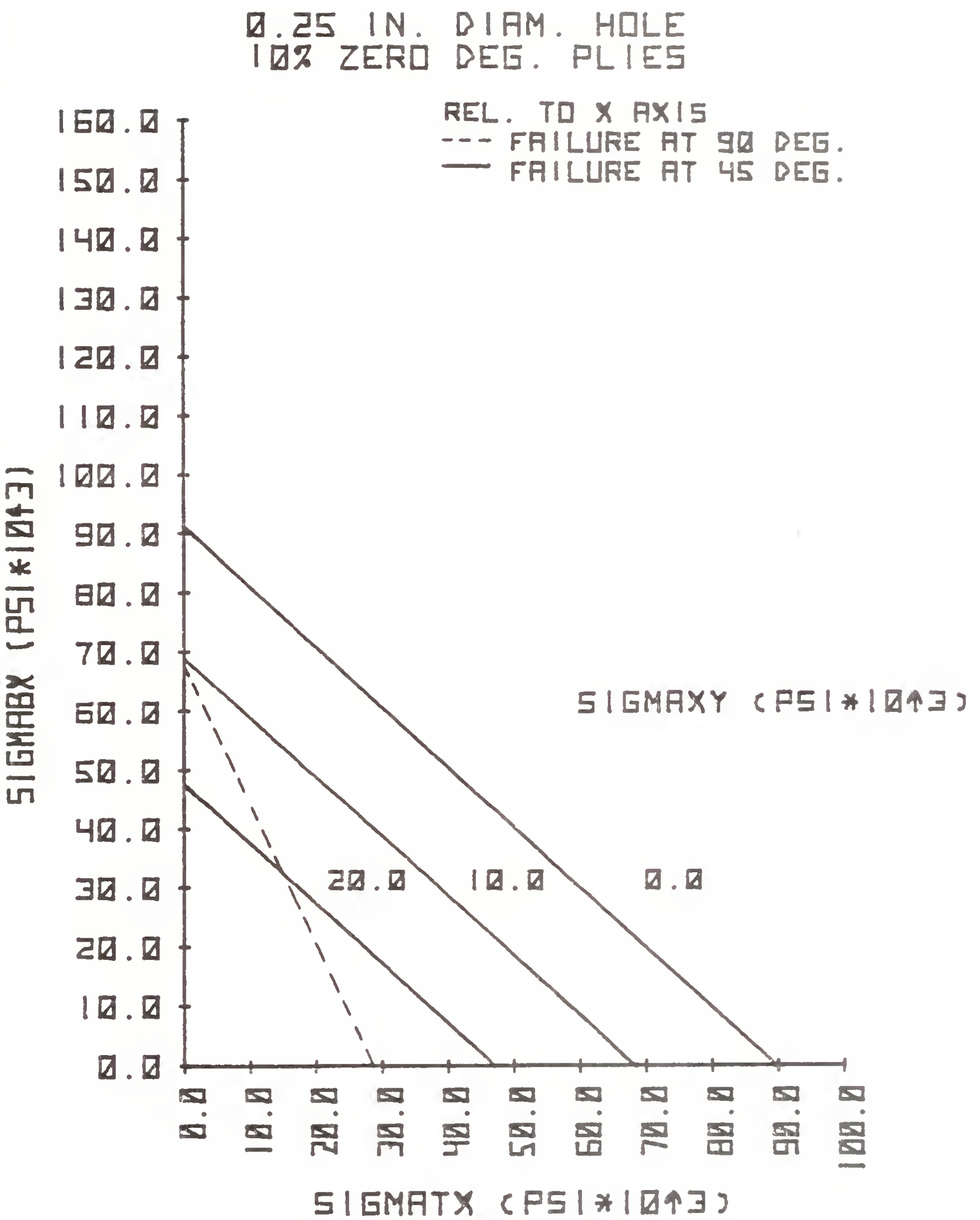


FIGURE 6. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

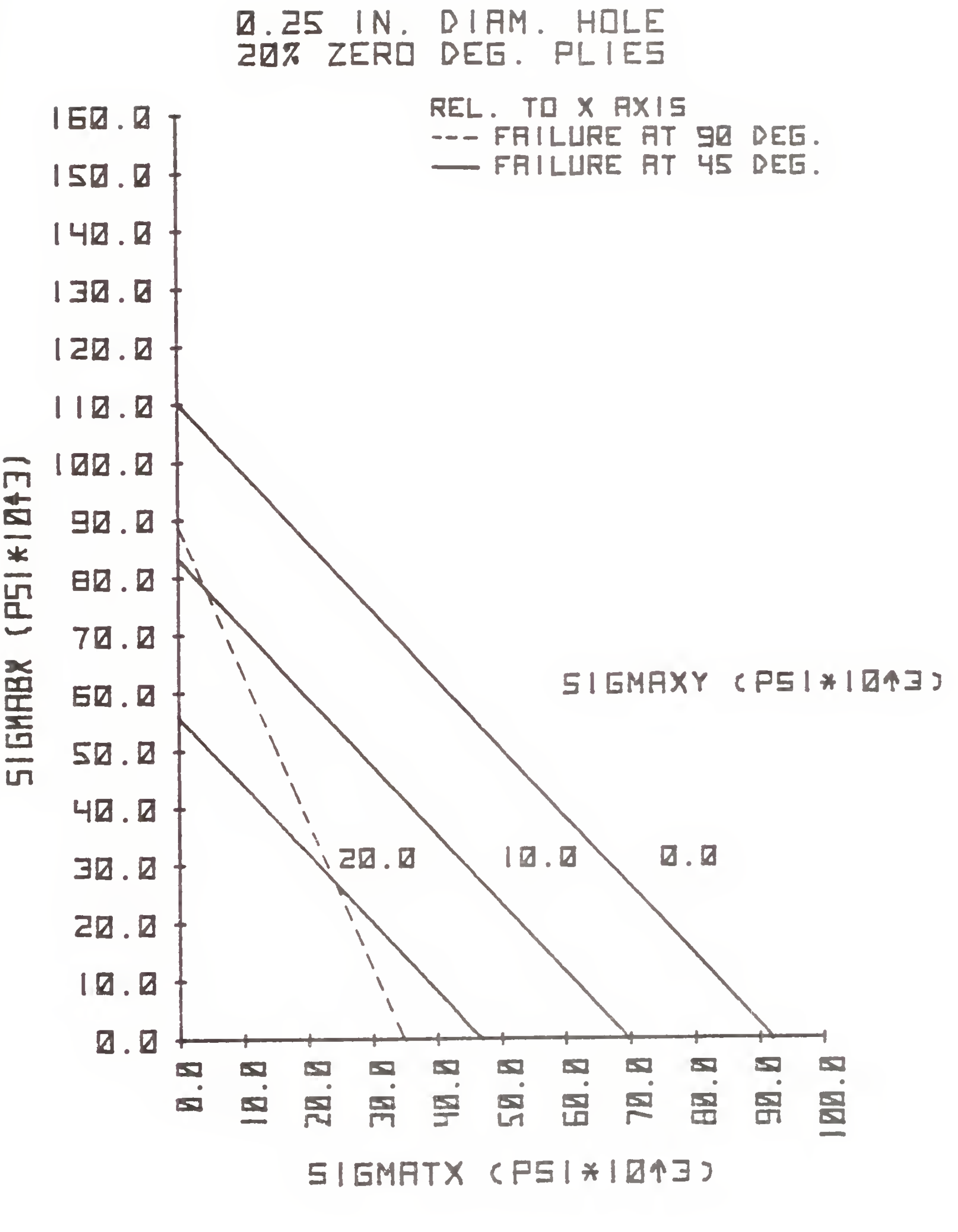


FIGURE 7. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

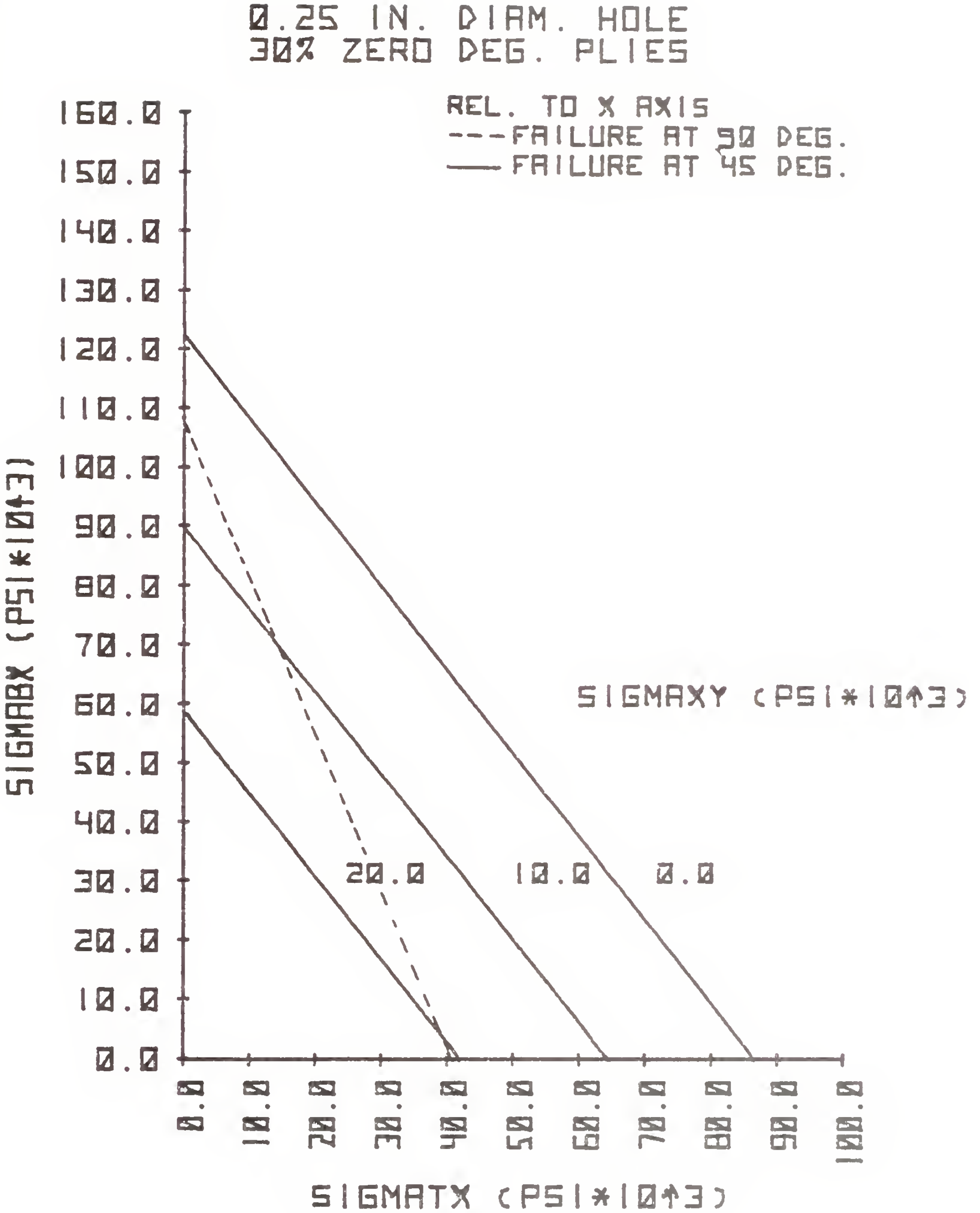


FIGURE 8. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

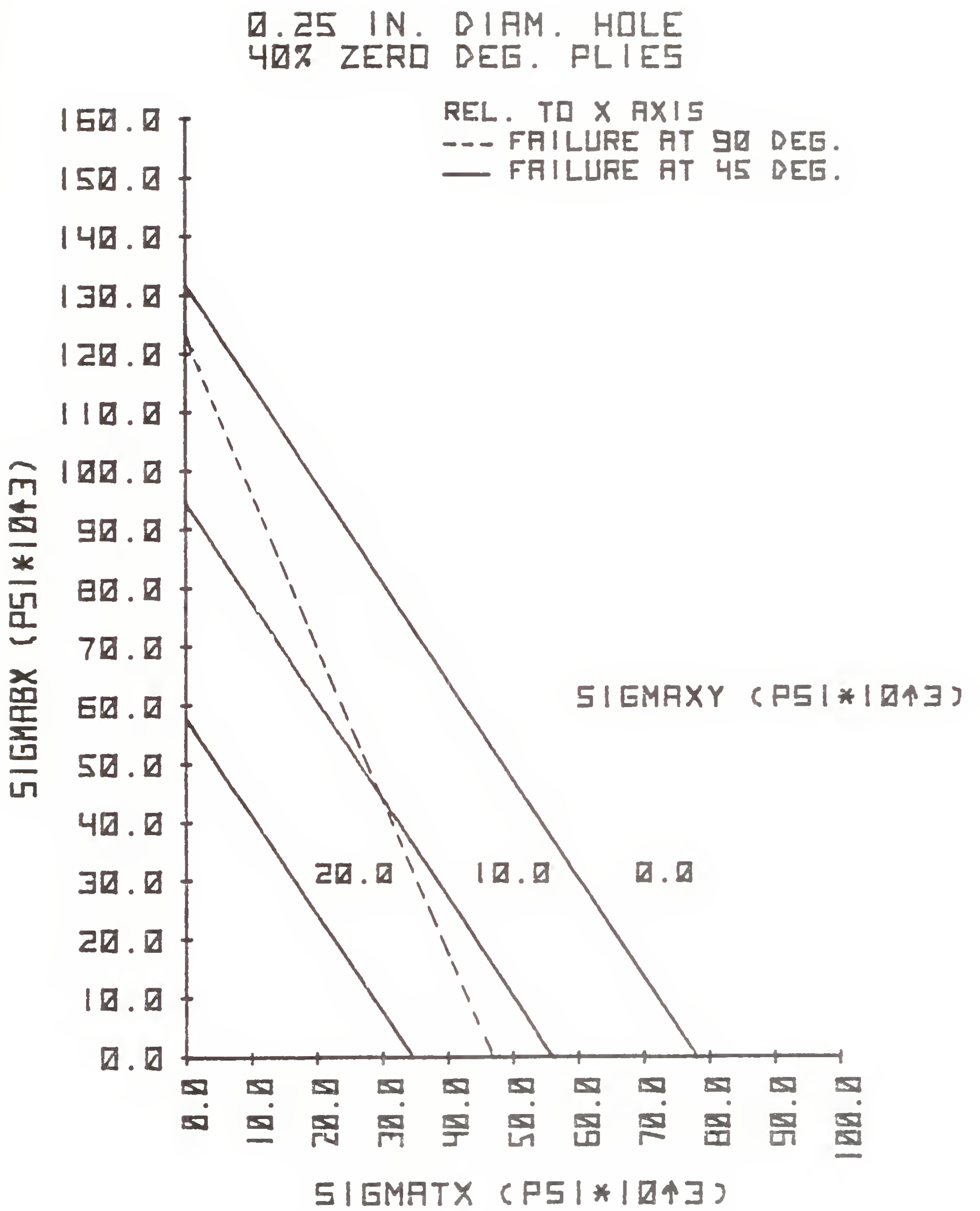


FIGURE 9. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

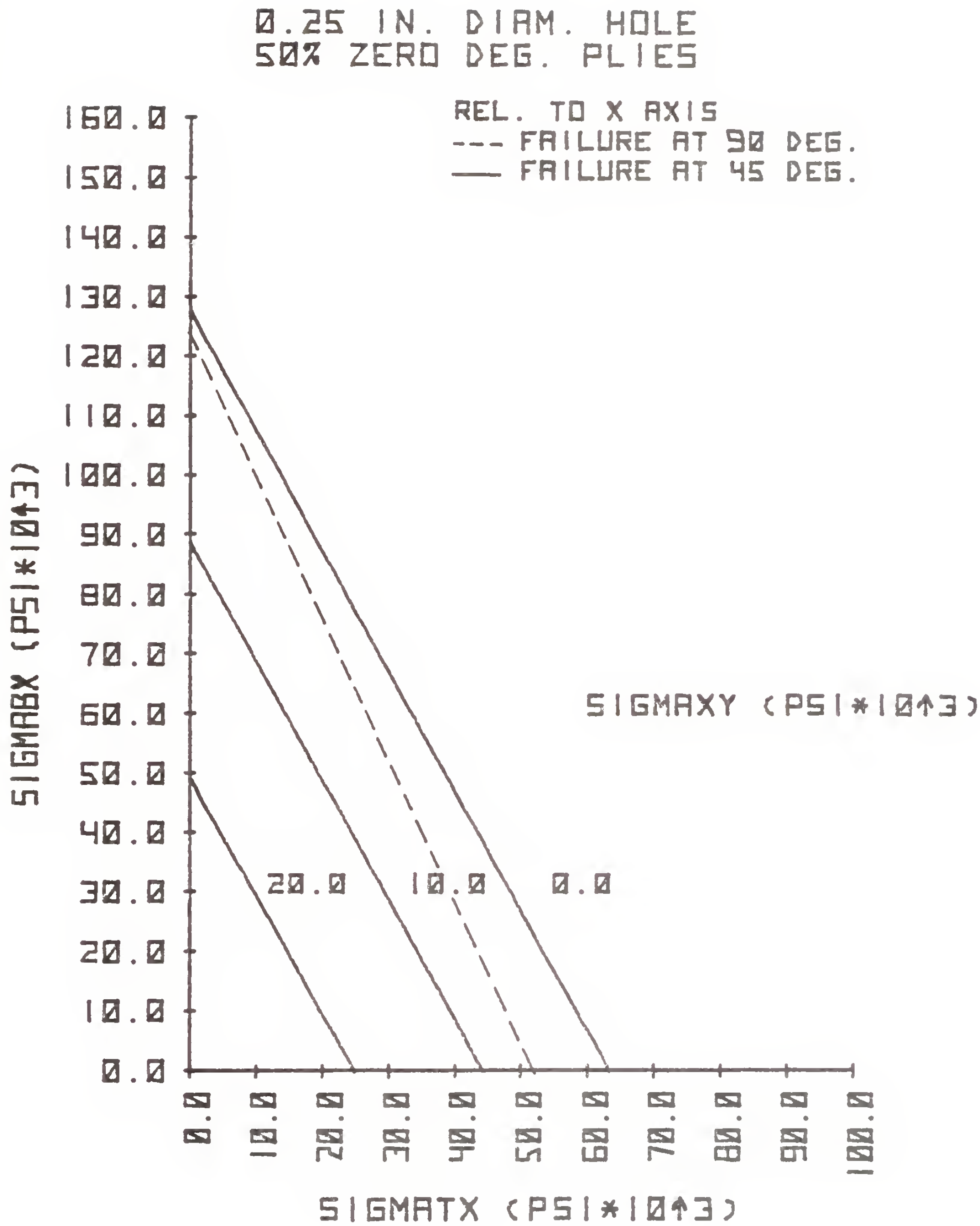


FIGURE 10. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

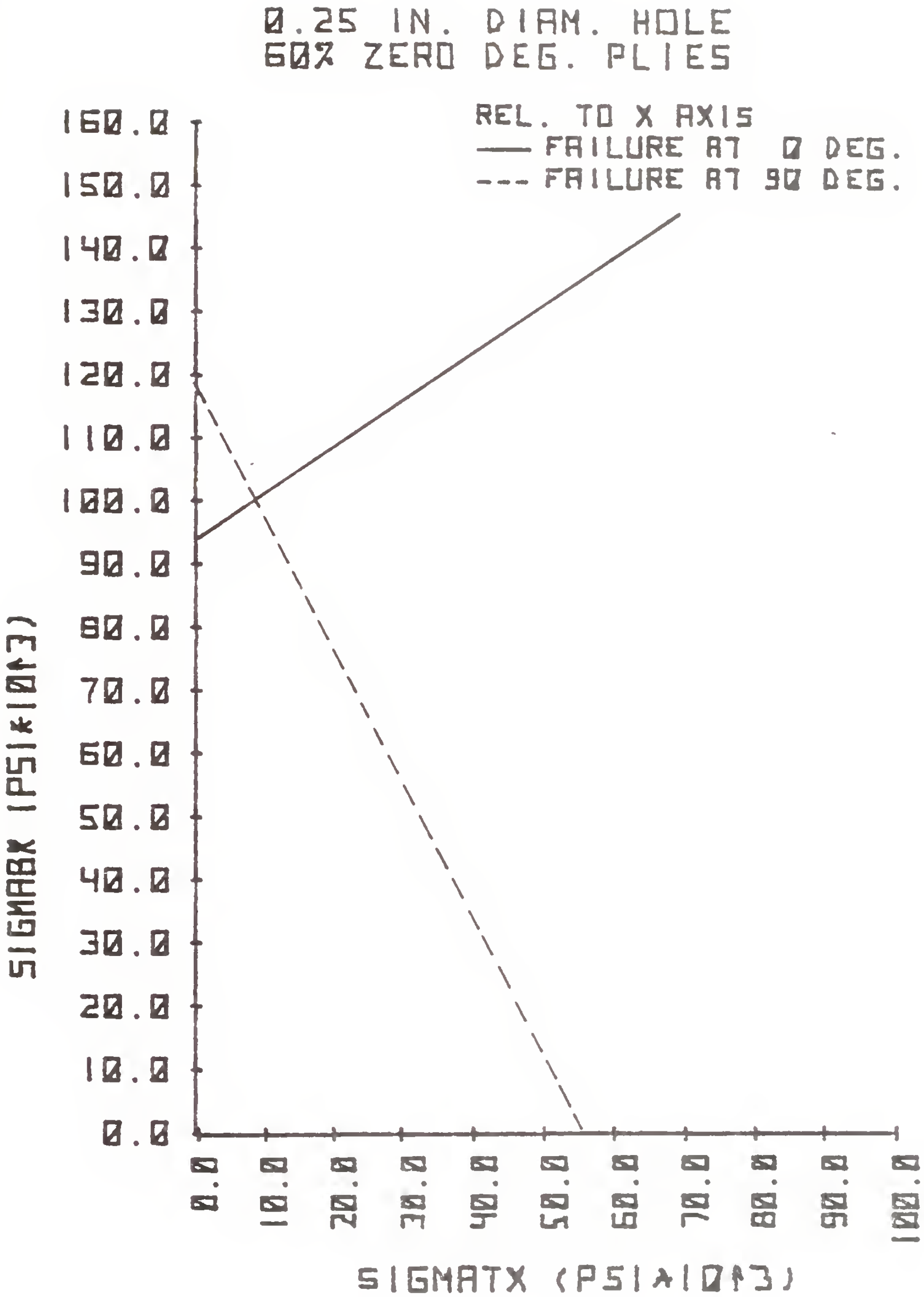


FIGURE 11. ULTIMATE STRESS INTERACTION CURVE FOR A ONE IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

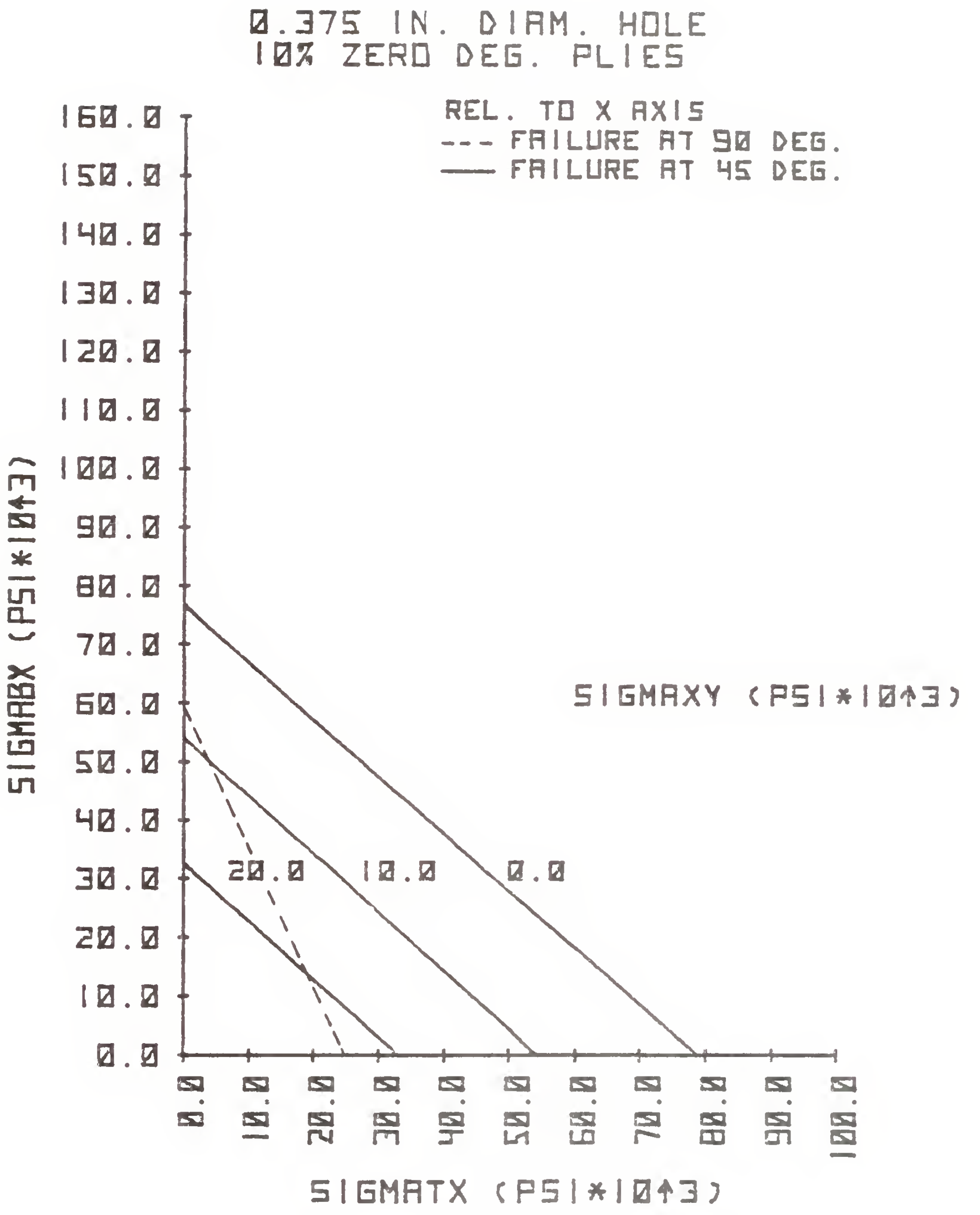


FIGURE 12. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

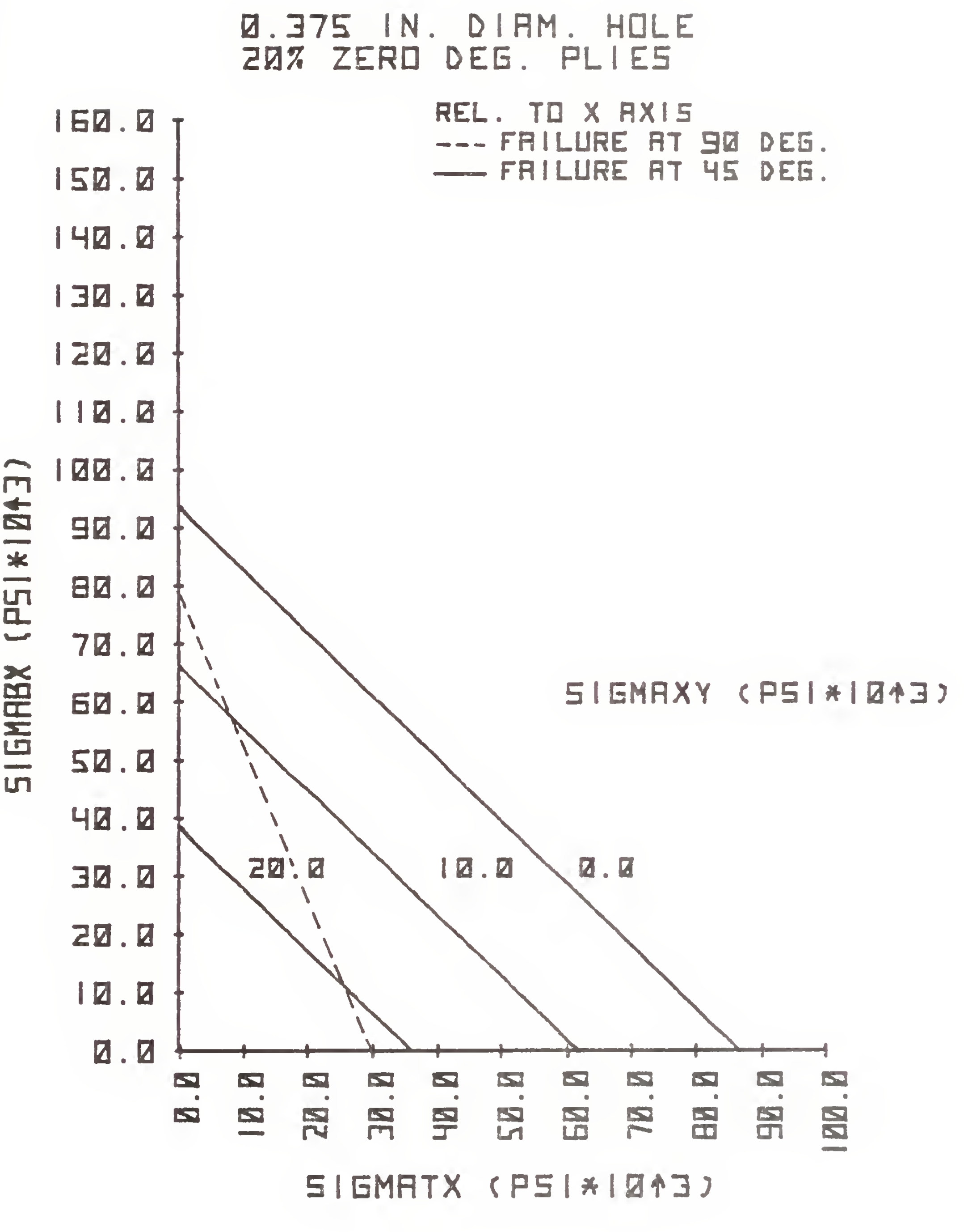


FIGURE 13. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

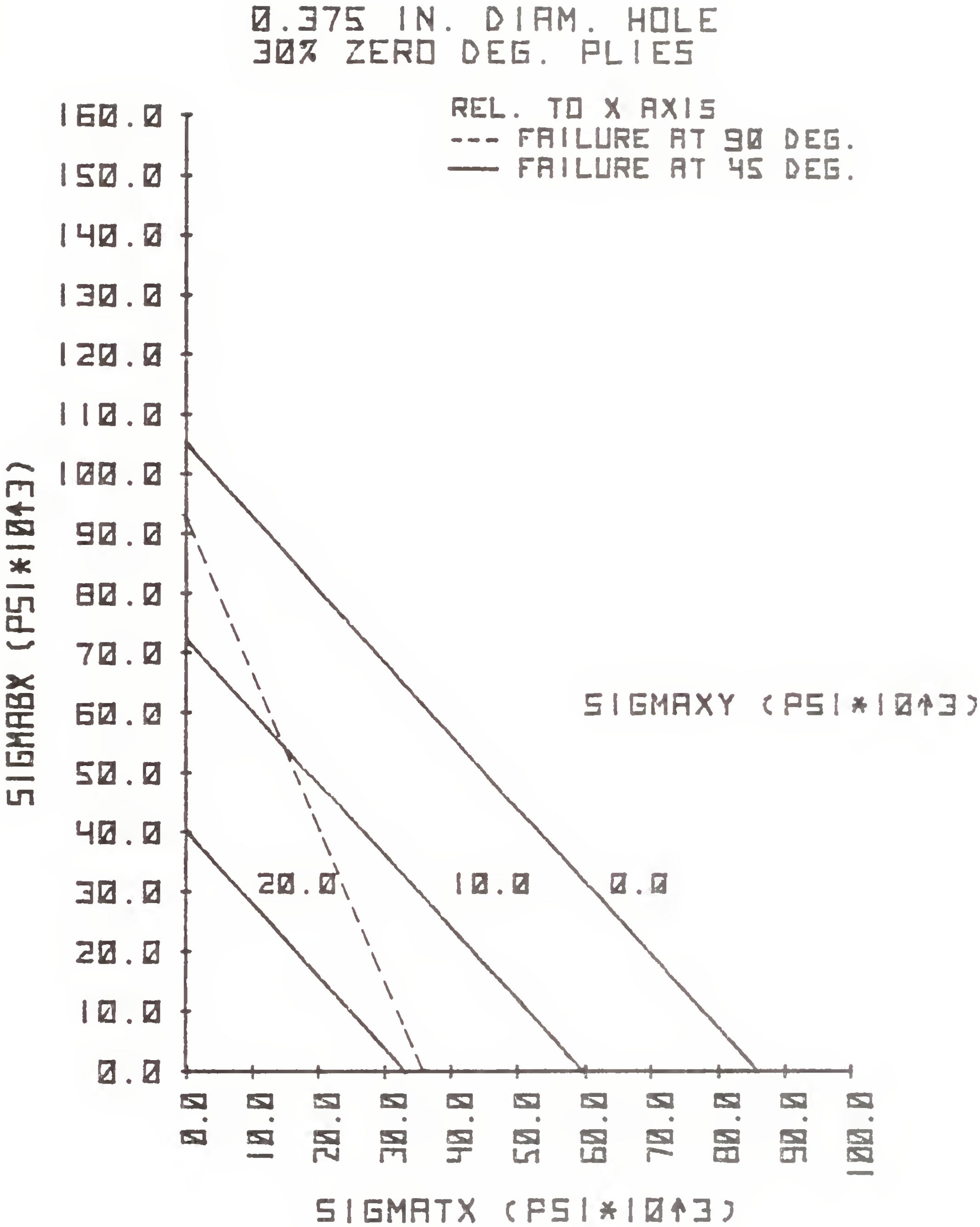


FIGURE 14. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

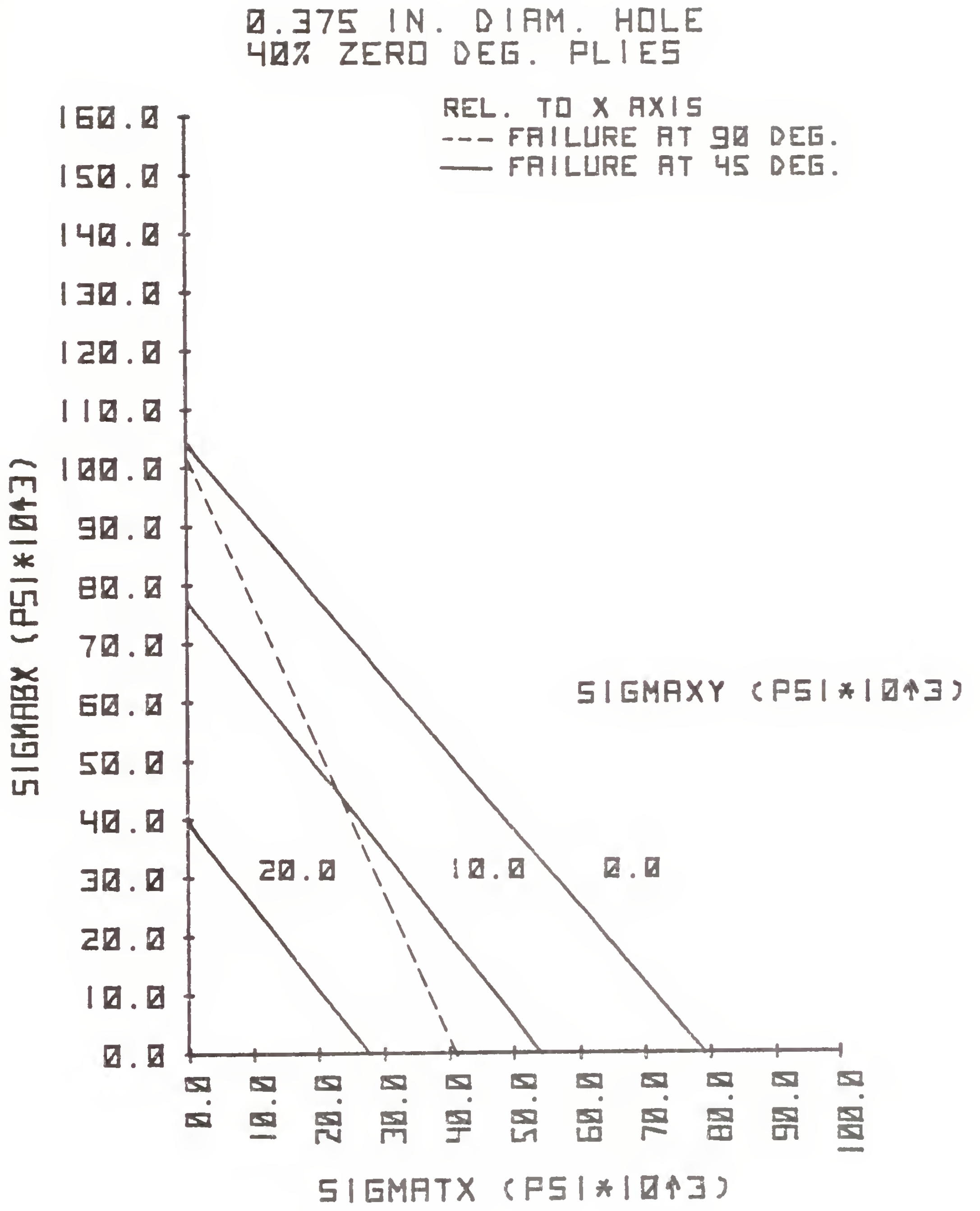


FIGURE 15. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

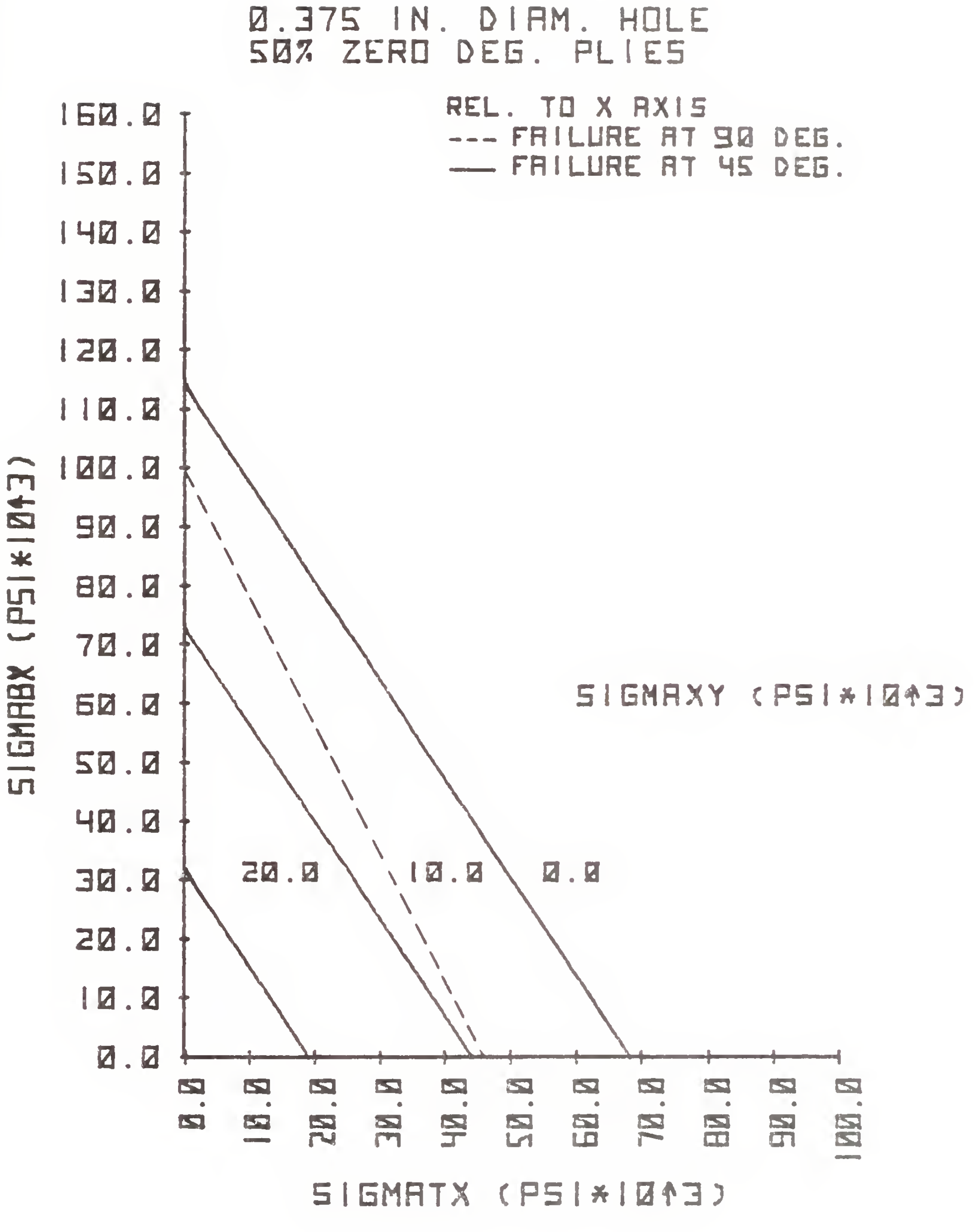


FIGURE 16. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

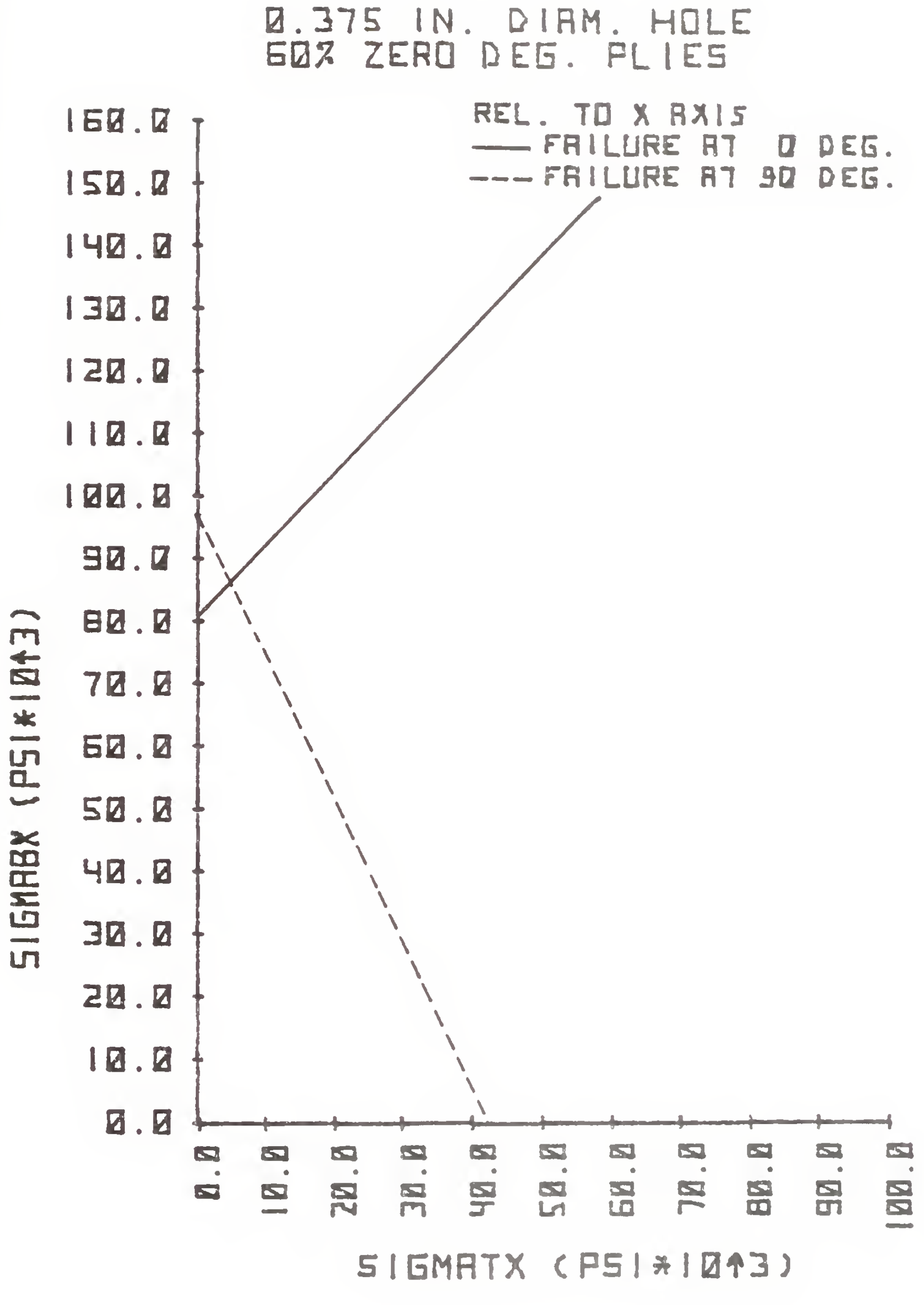


FIGURE 17. ULTIMATE STRESS INTERACTION CURVE FOR A 1.5 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

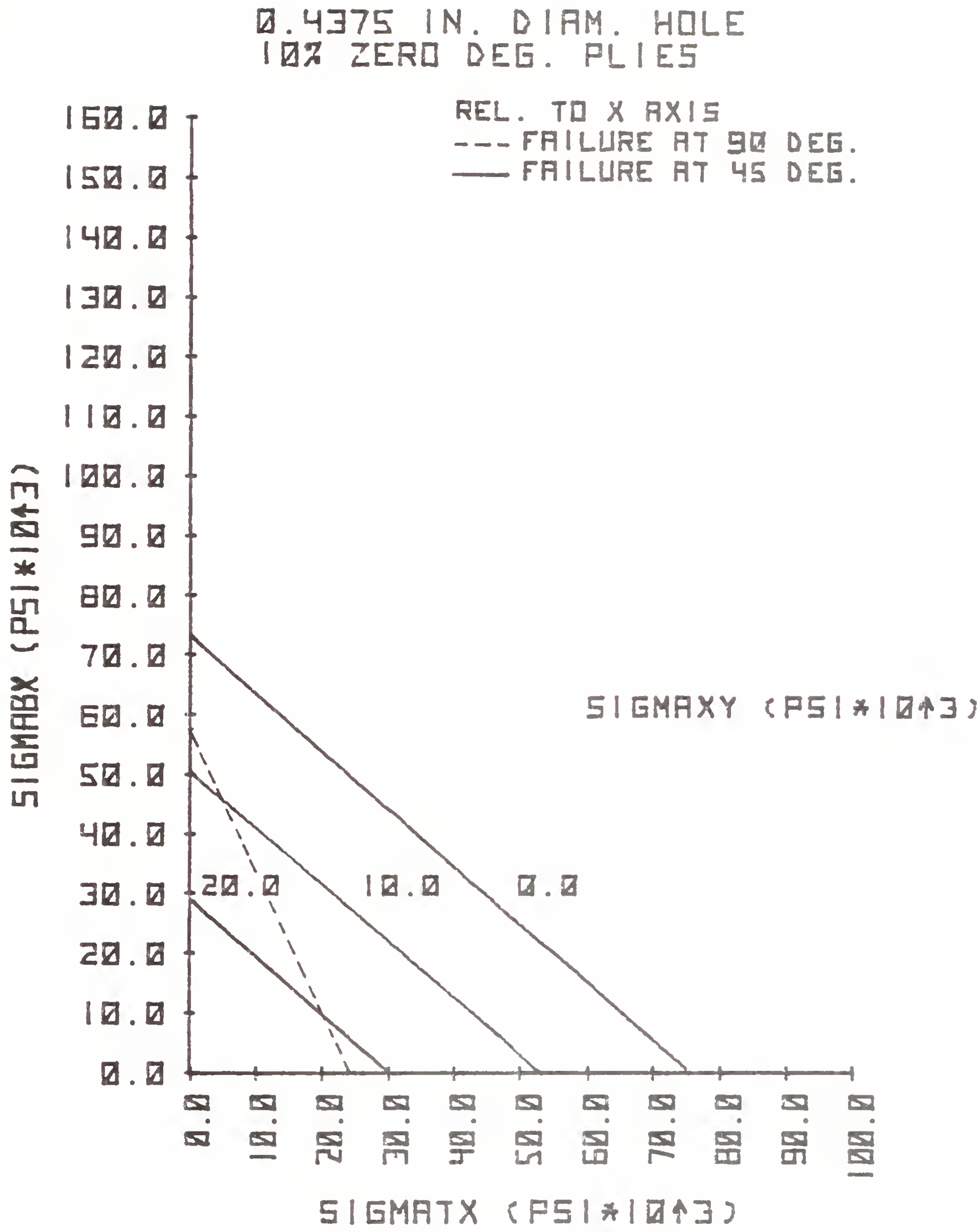


FIGURE 18. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

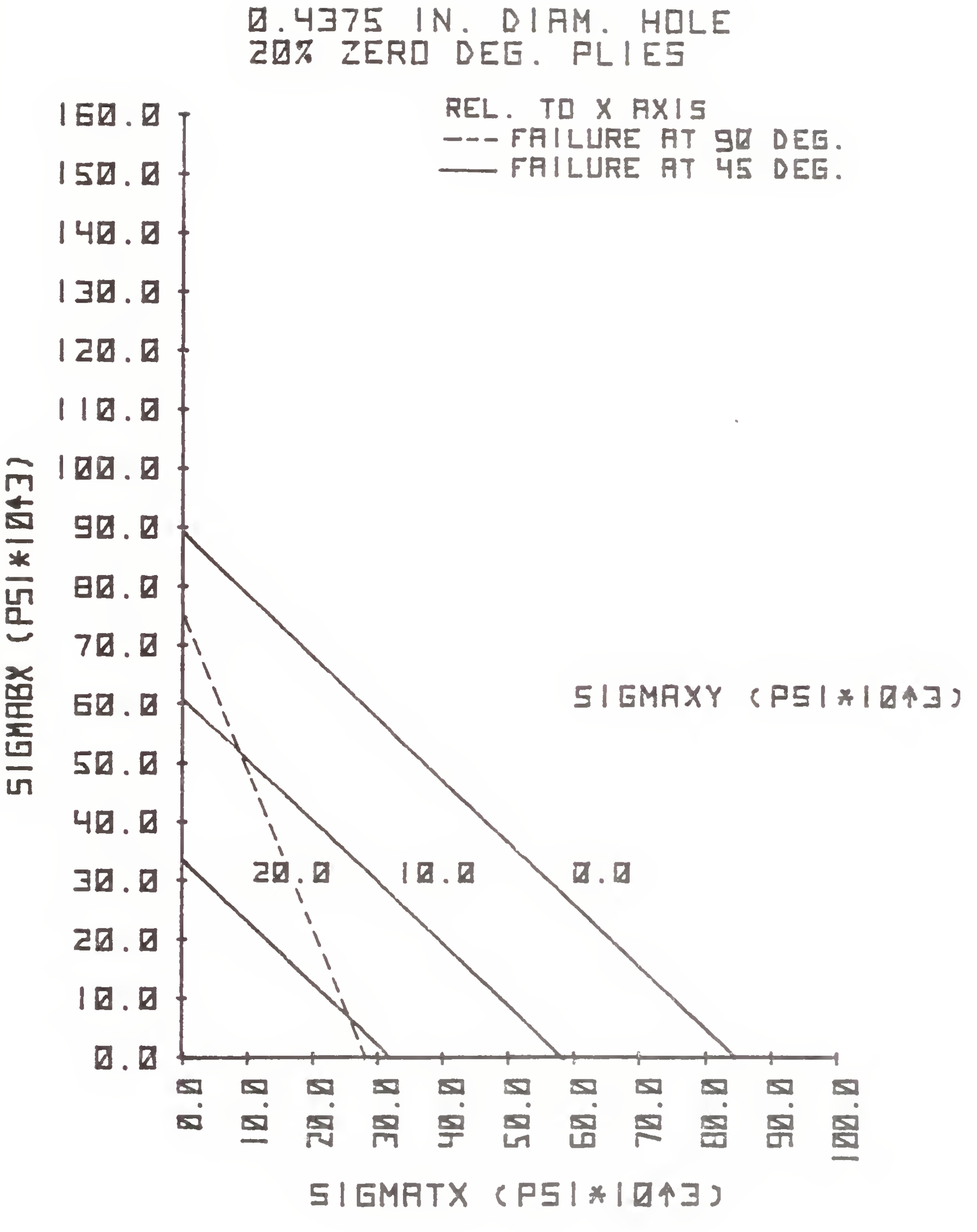


FIGURE 19. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

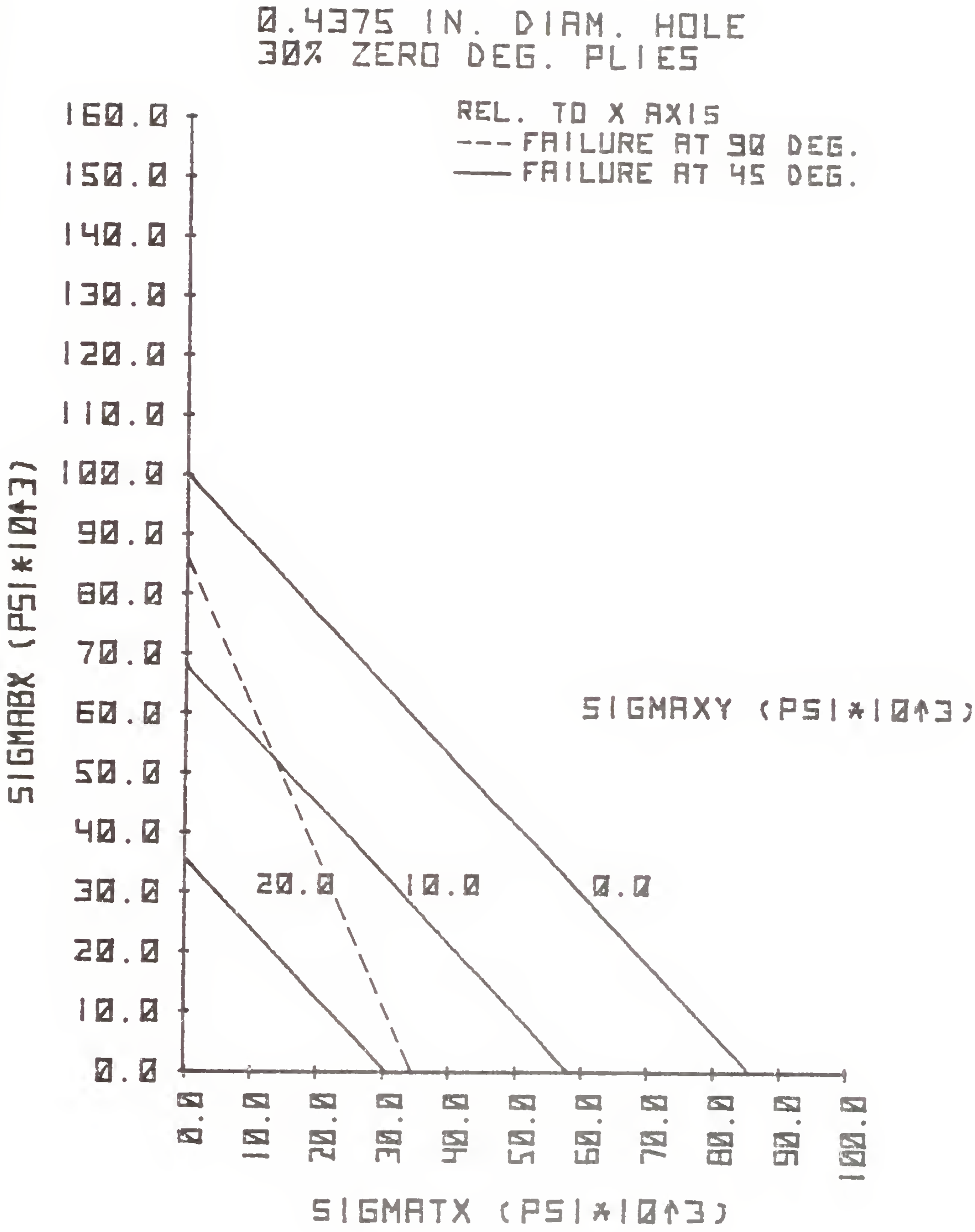


FIGURE 20. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

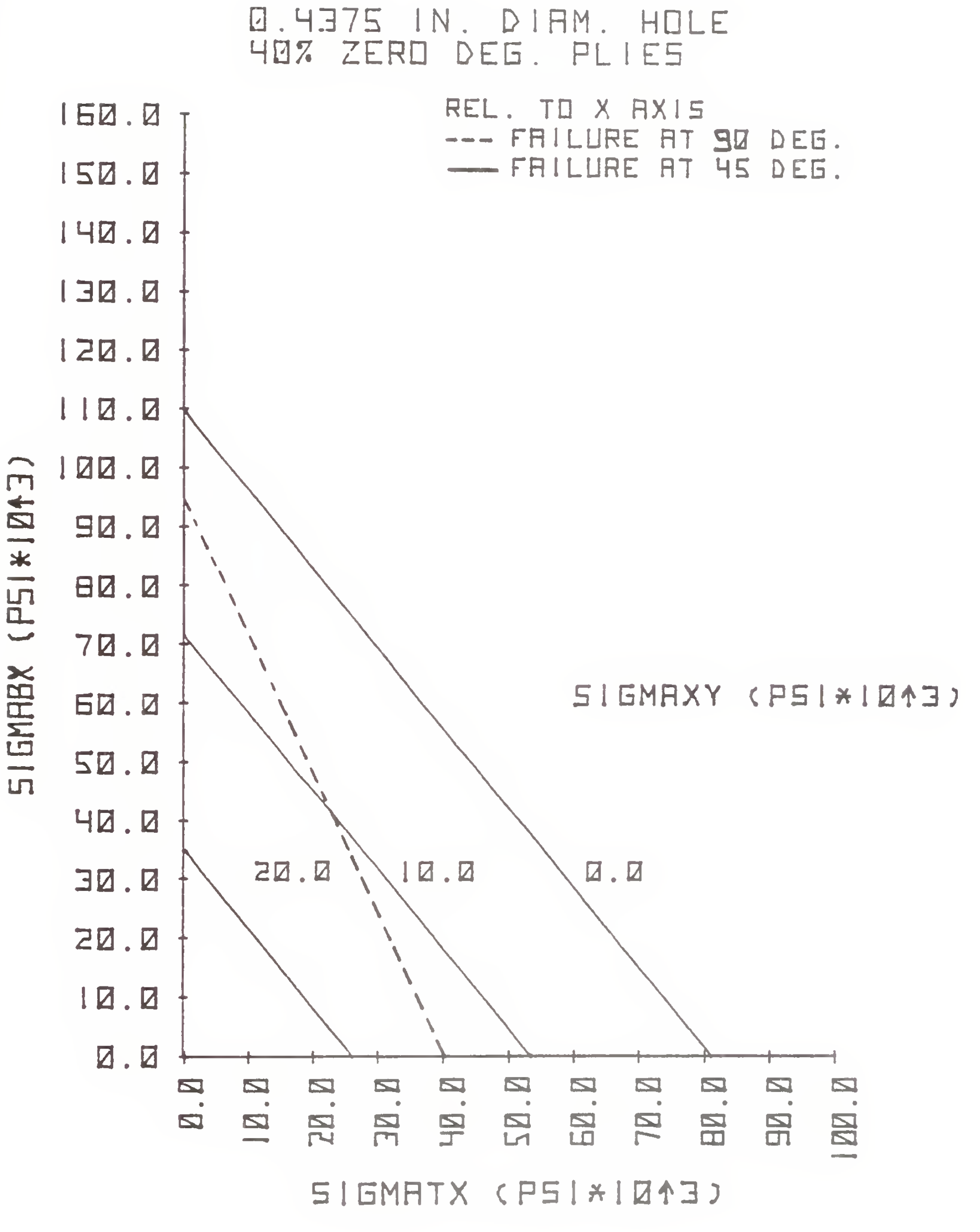


FIGURE 21. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

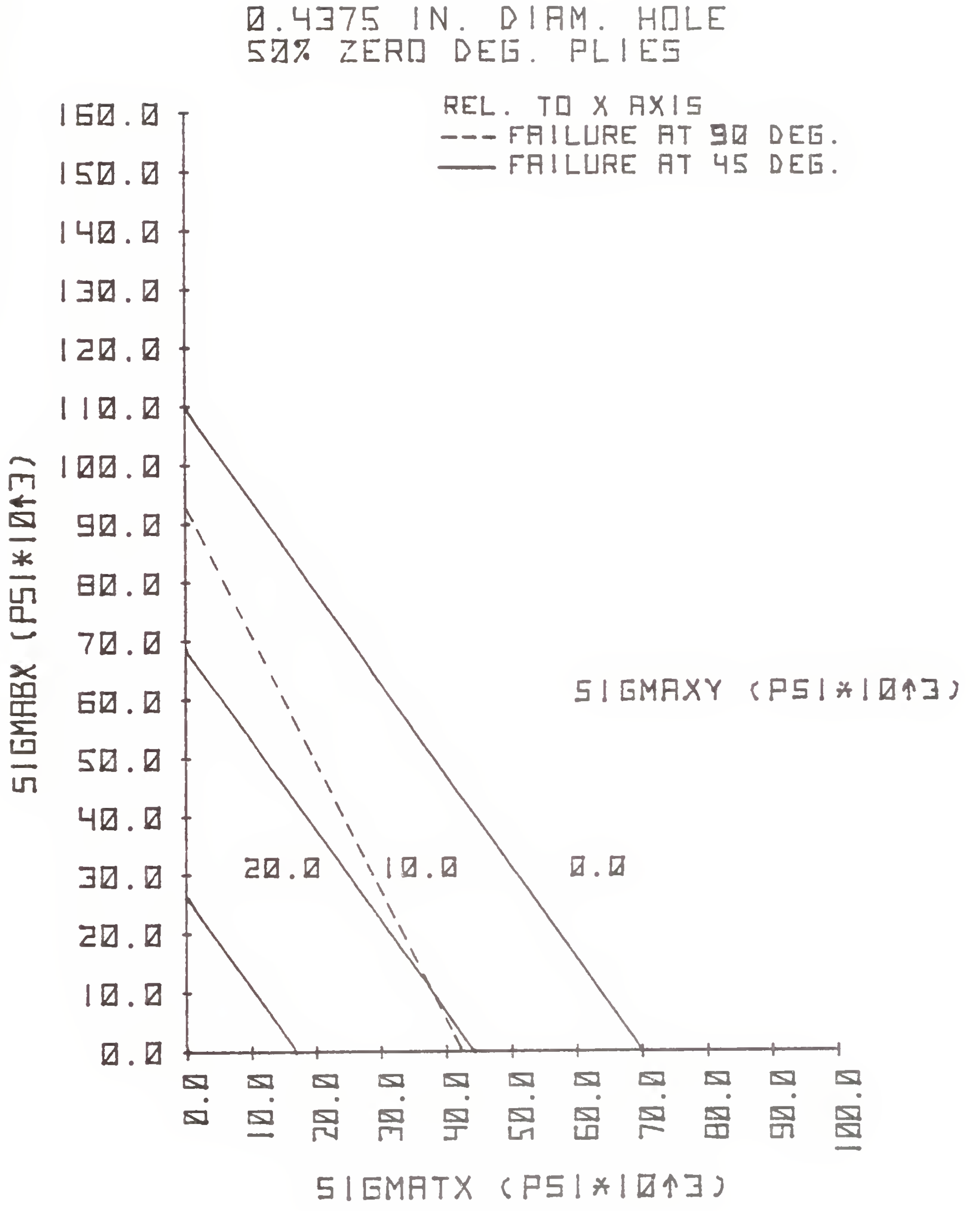


FIGURE 22. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

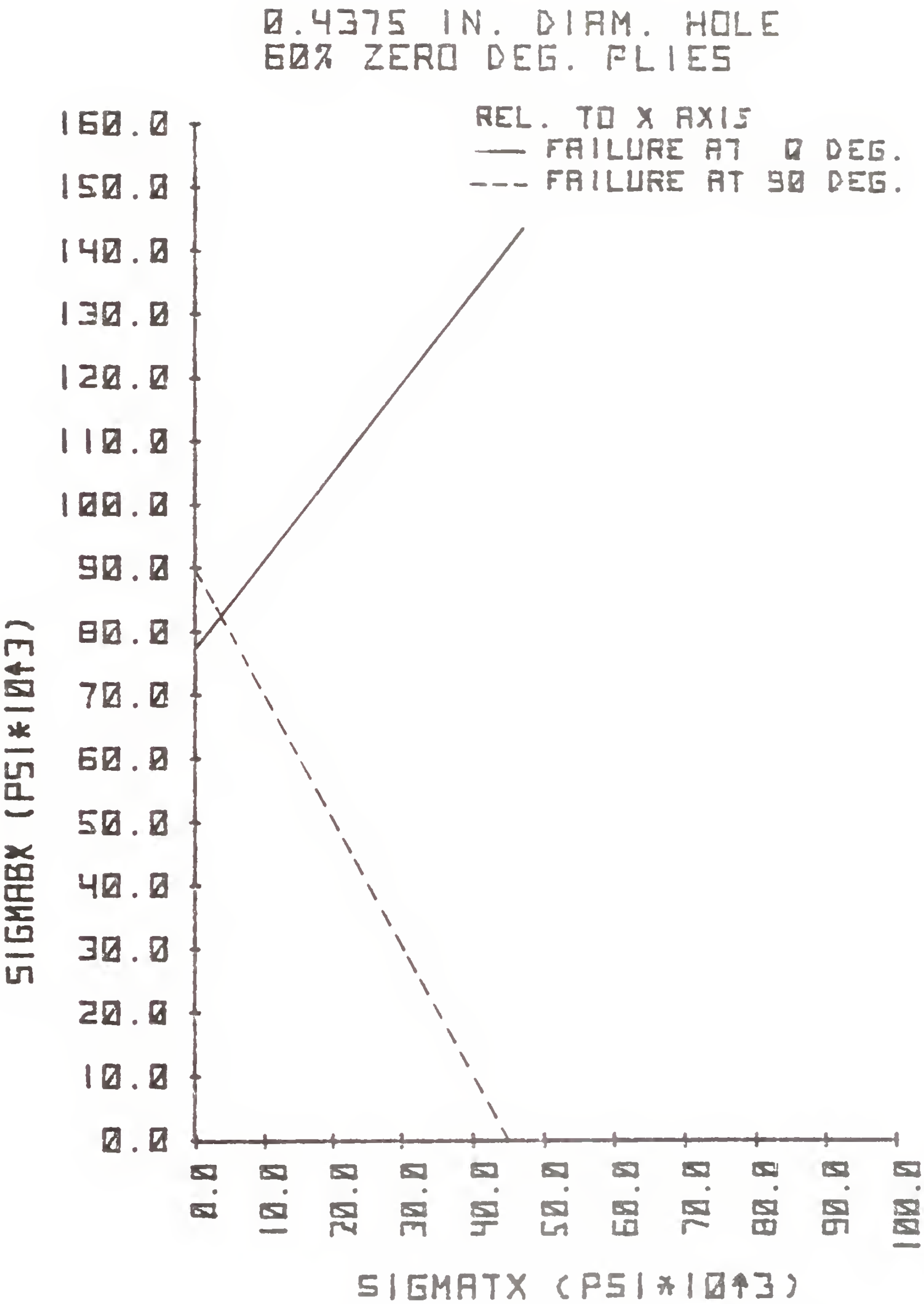


FIGURE 23. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

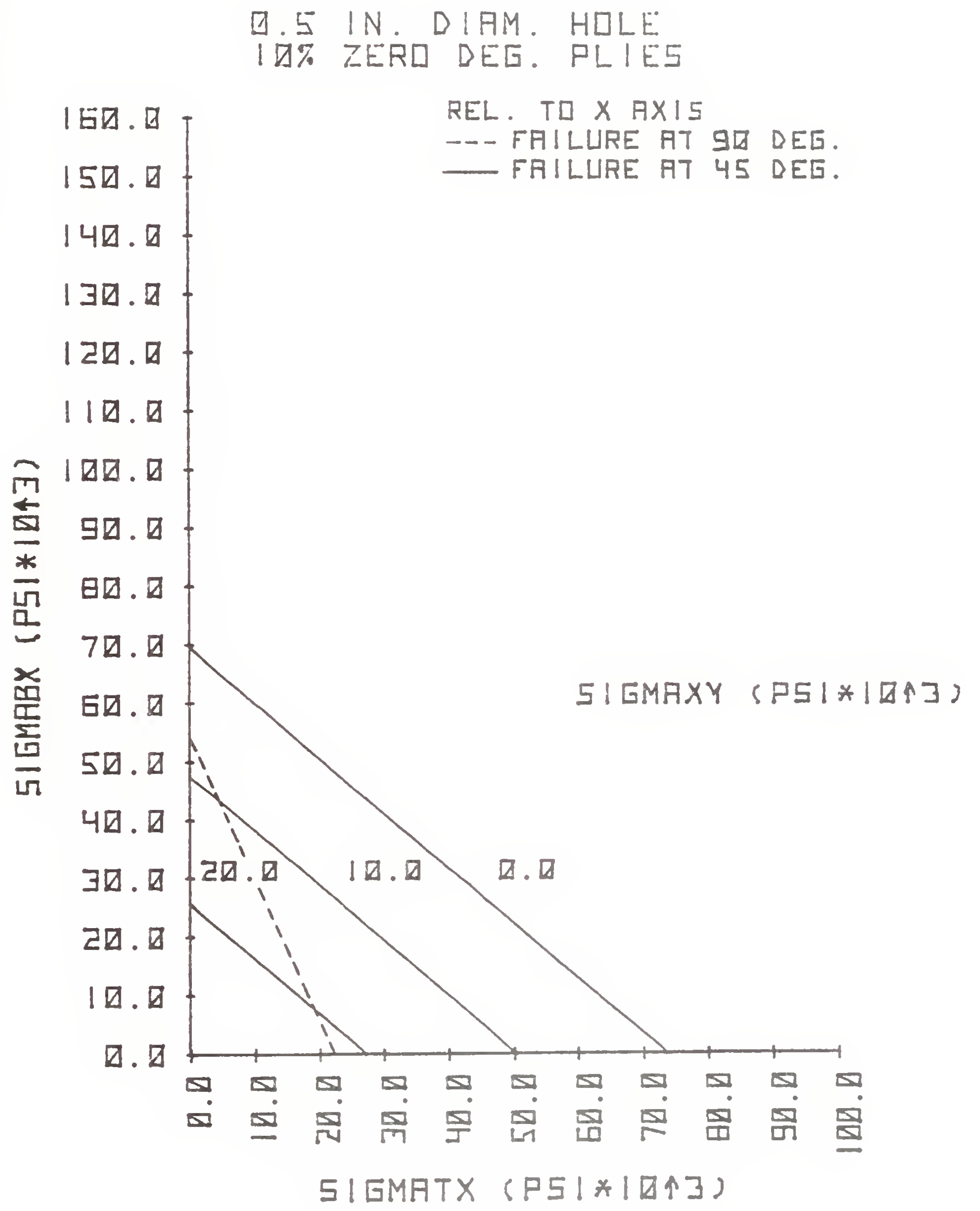


FIGURE 24. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 10 PER CENT ZERO DEGREE PLIES

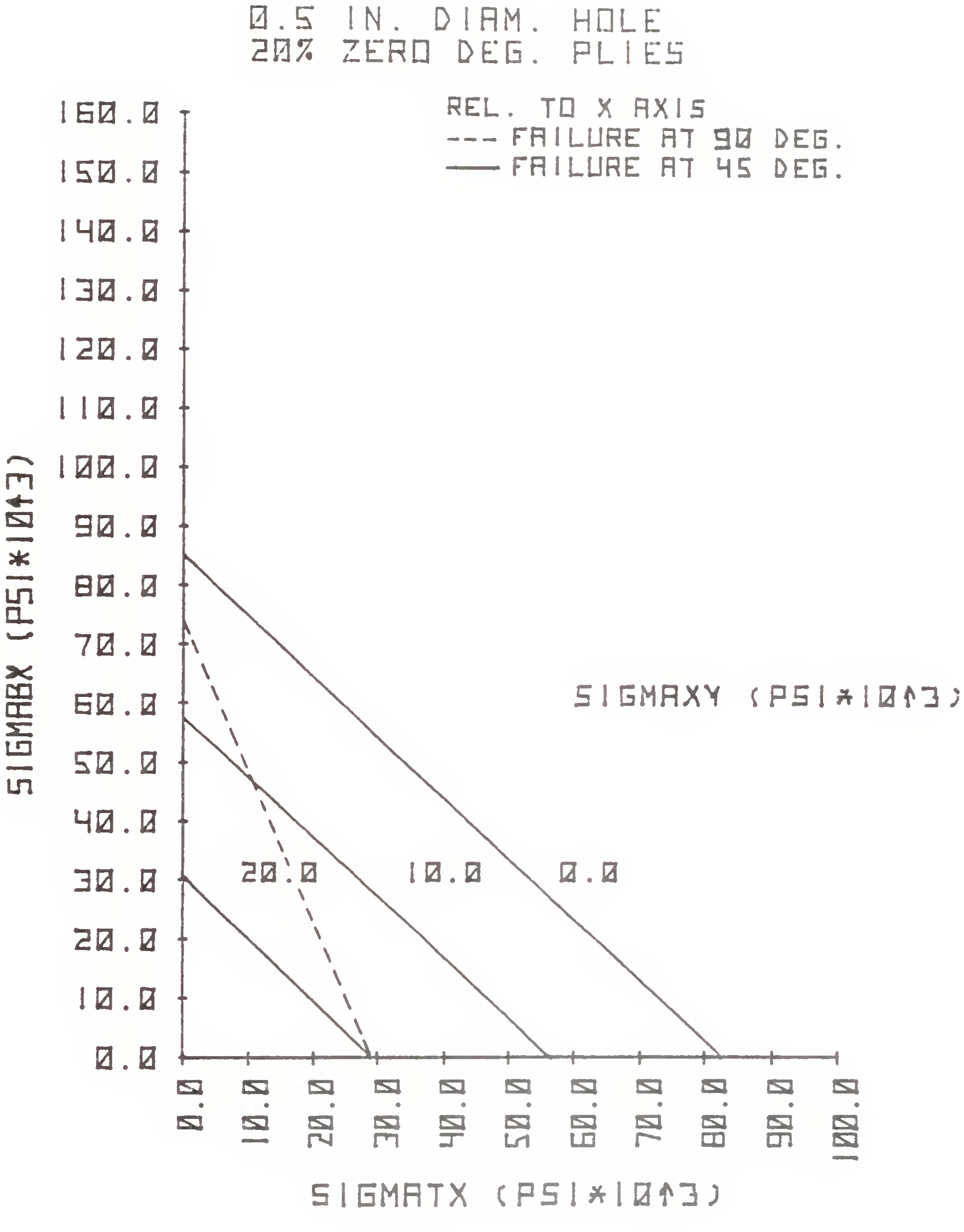


FIGURE 25. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 20 PER CENT ZERO DEGREE PLIES

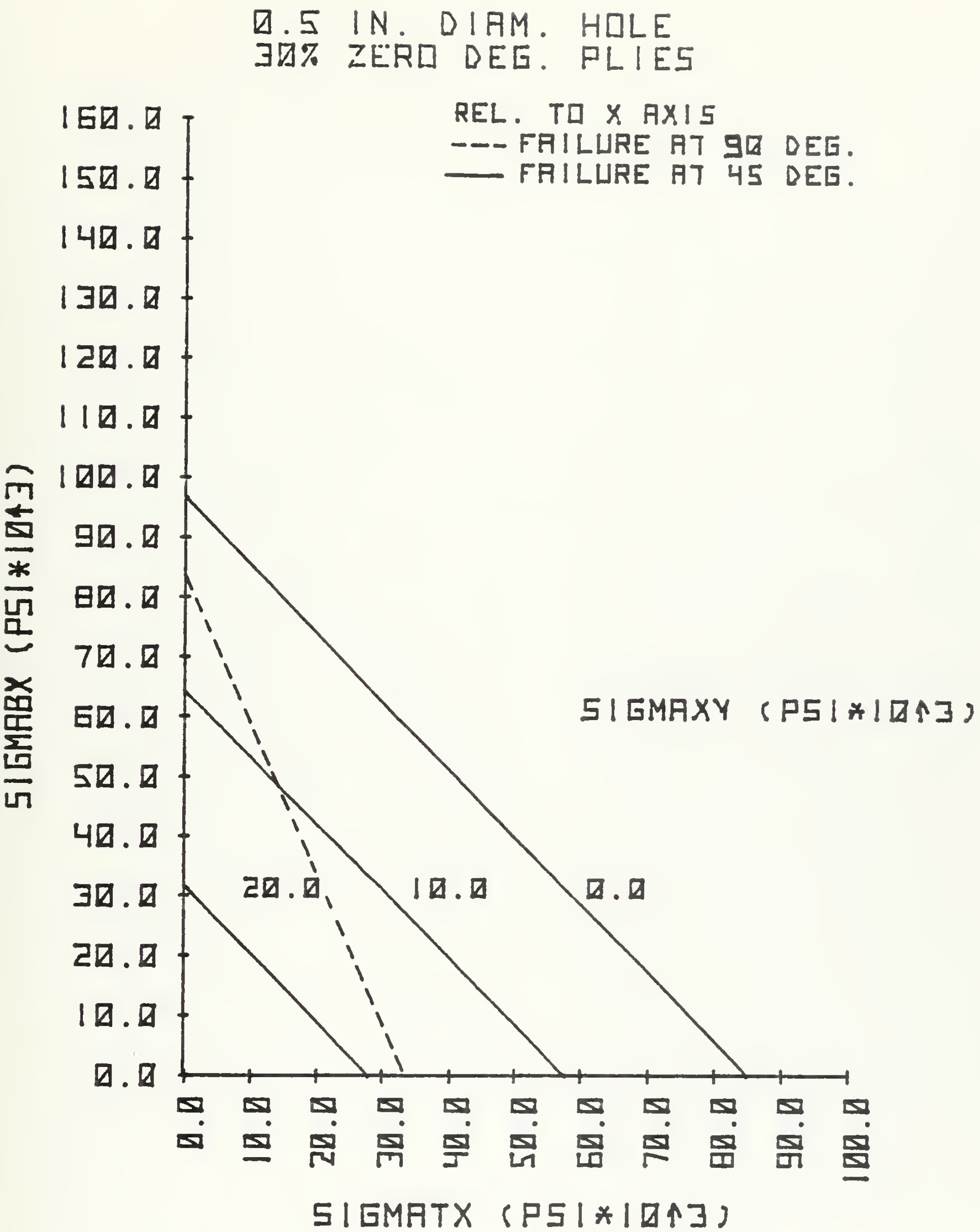


FIGURE 26. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 30 PER CENT ZERO DEGREE PLIES

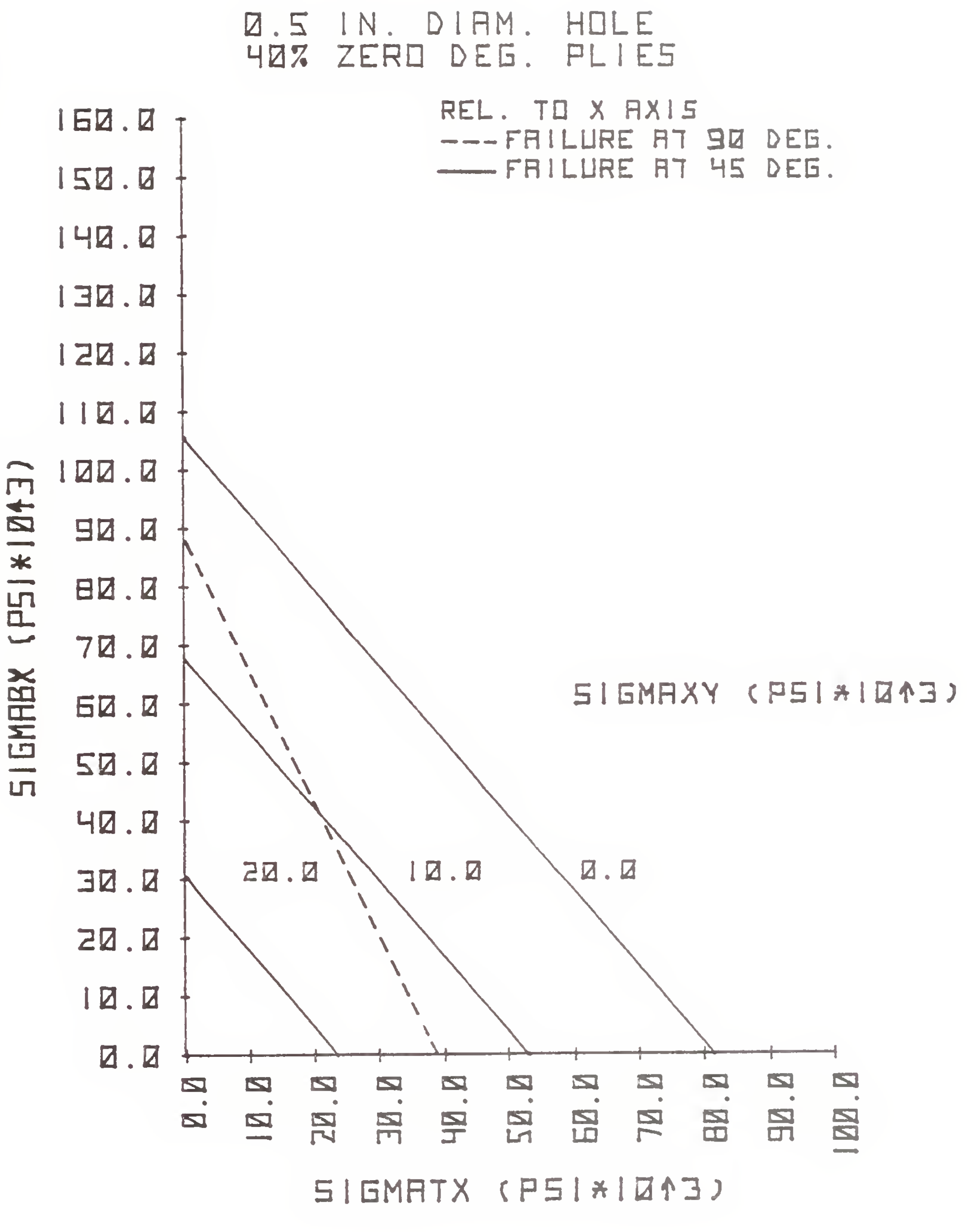


FIGURE 27. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 40 PER CENT ZERO DEGREE PLIES

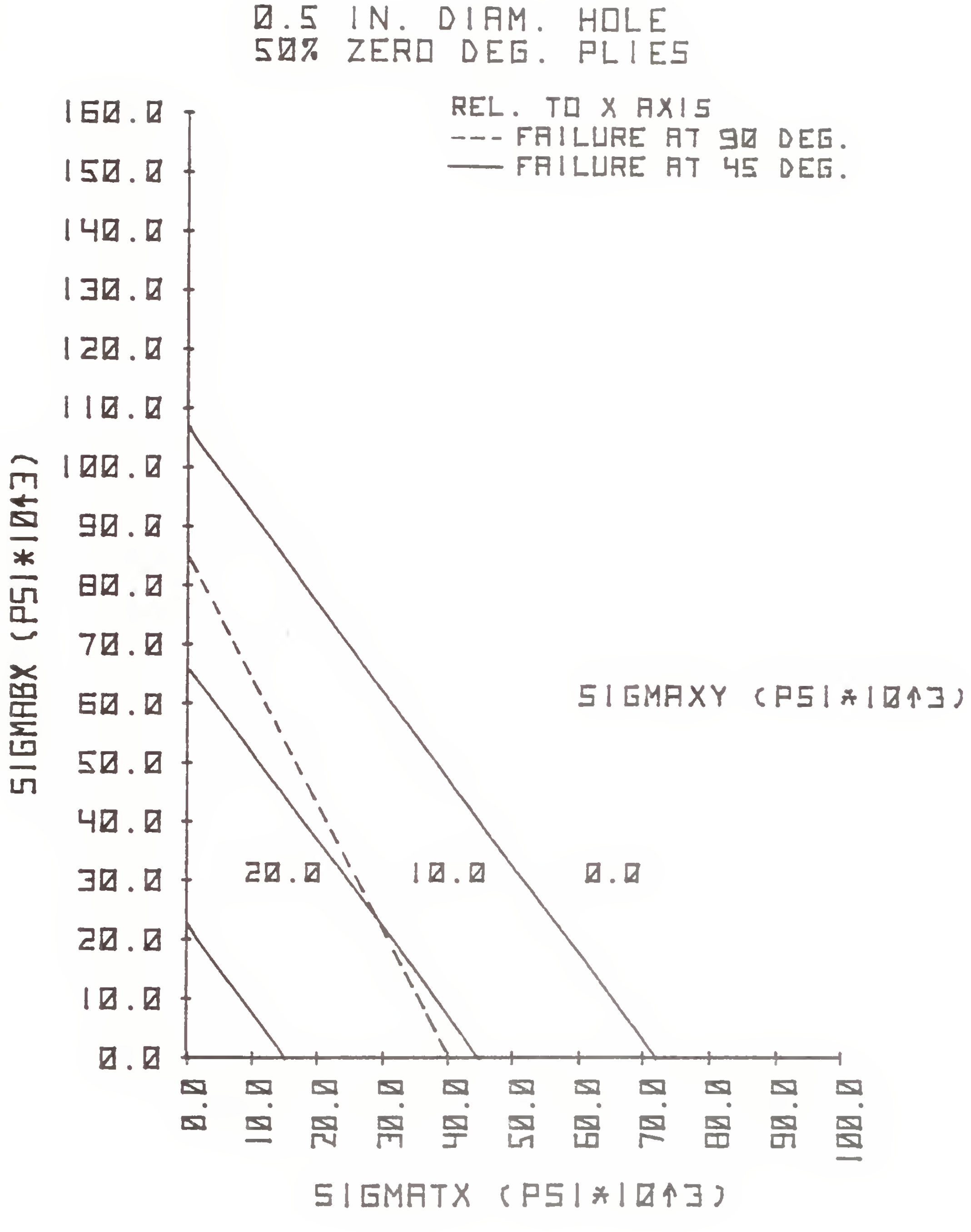


FIGURE 28. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45] MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 50 PER CENT ZERO DEGREE PLIES

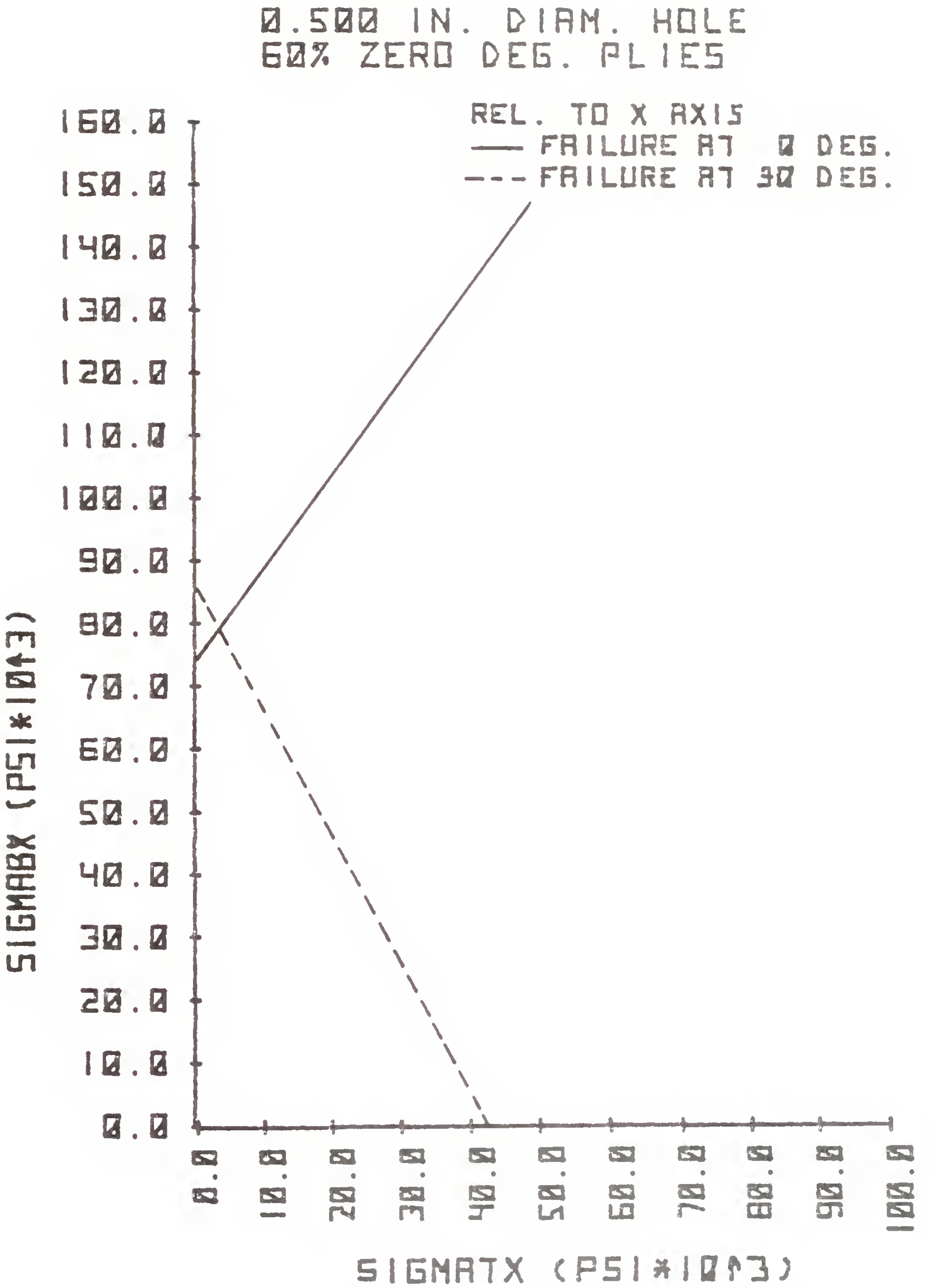


FIGURE 29. ULTIMATE STRESS INTERACTION CURVE FOR A 2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 $[0/\pm45]$ MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 60 PER CENT ZERO DEGREE PLIES

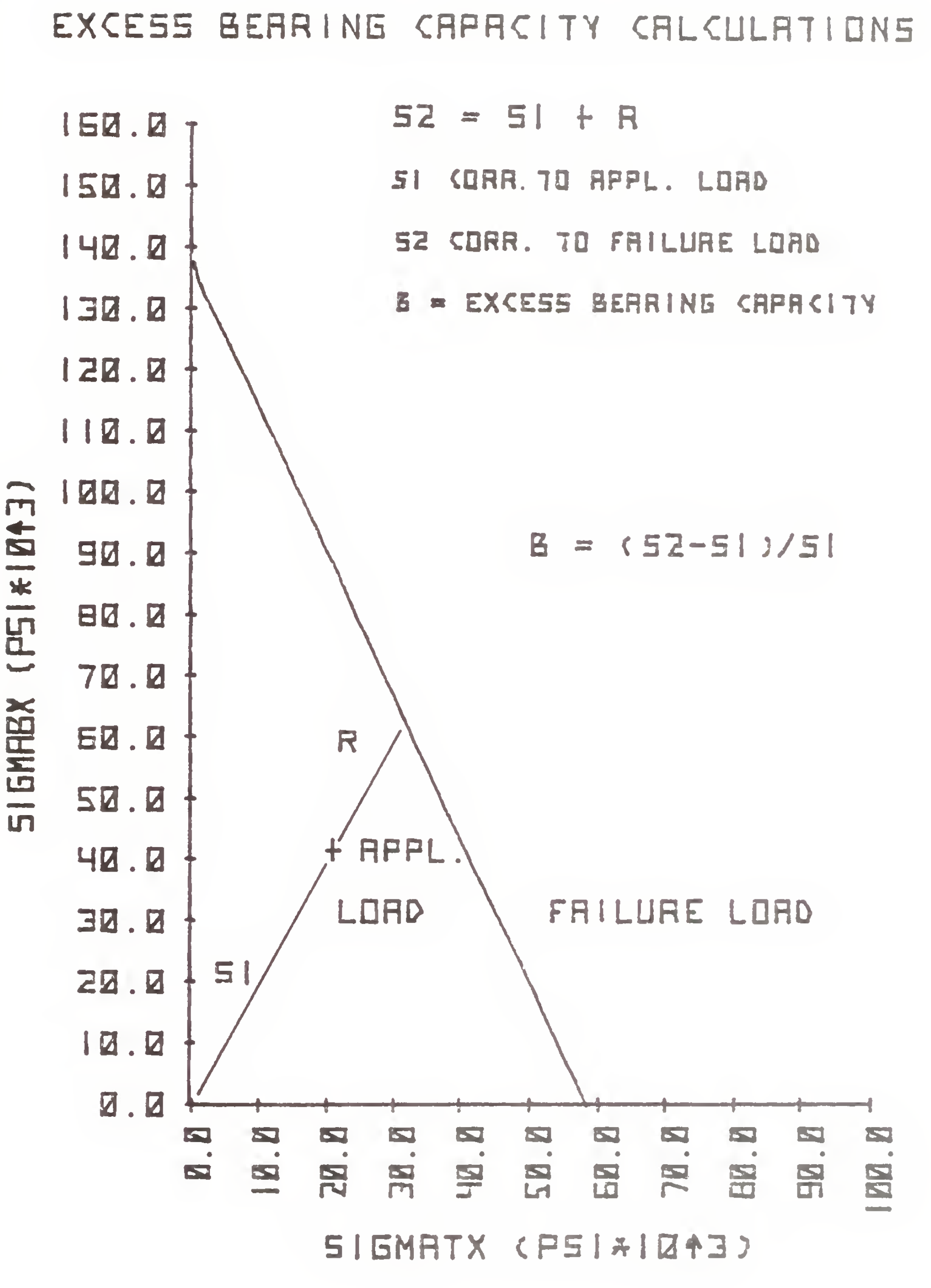


FIGURE 30. EXCESS BEARING CAPACITY CALCULATIONS

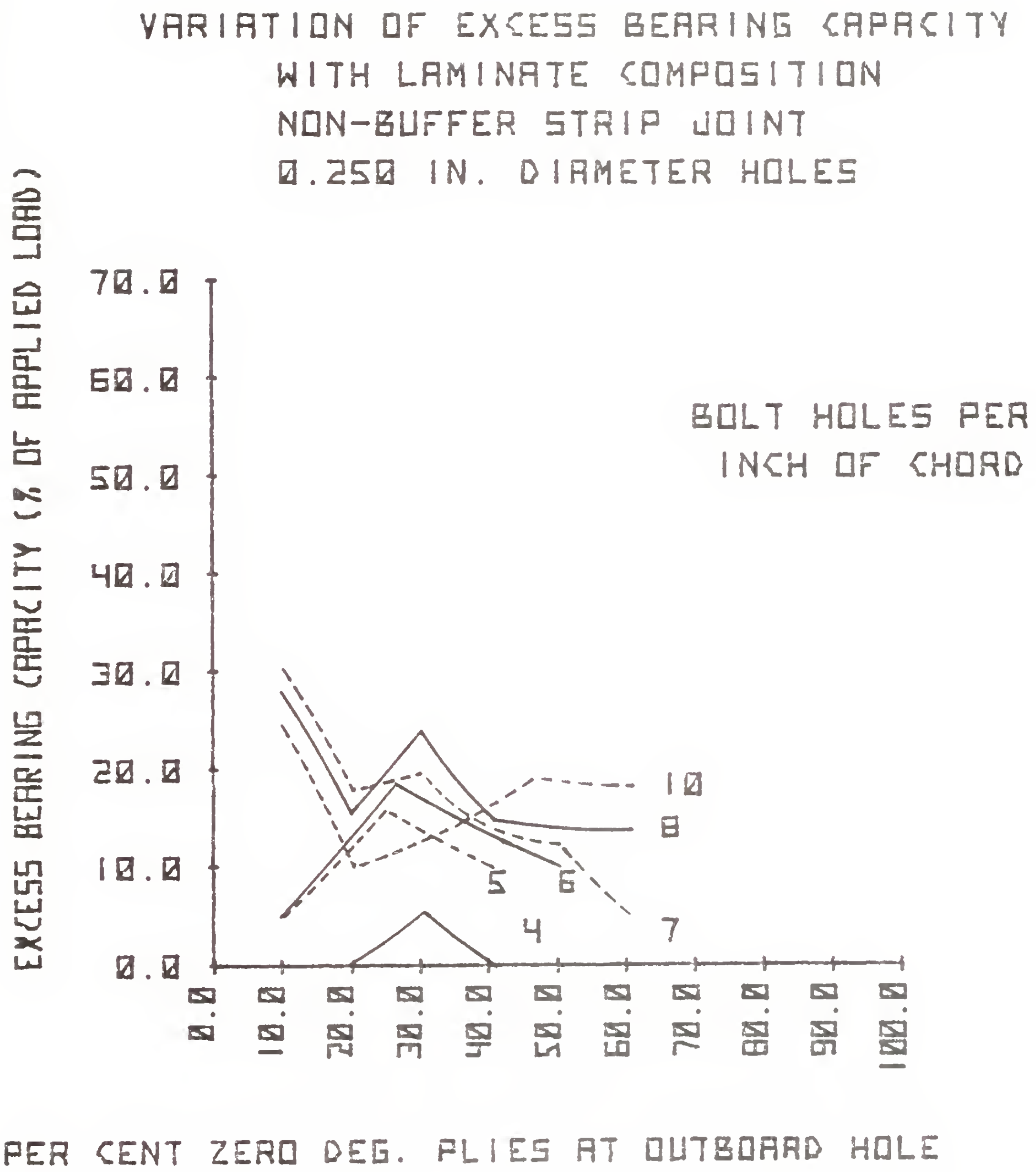


FIGURE 31. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

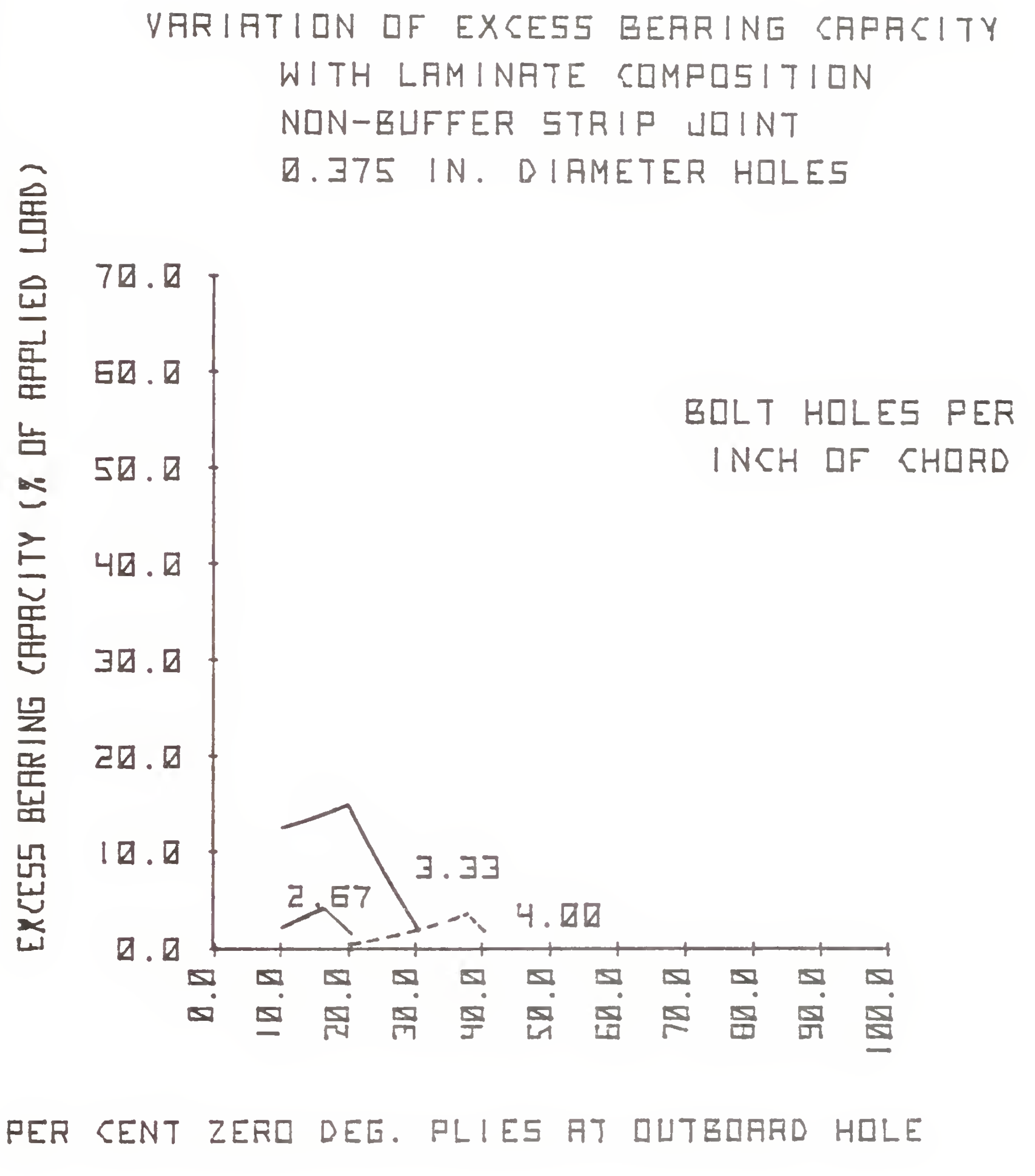


FIGURE 32. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES

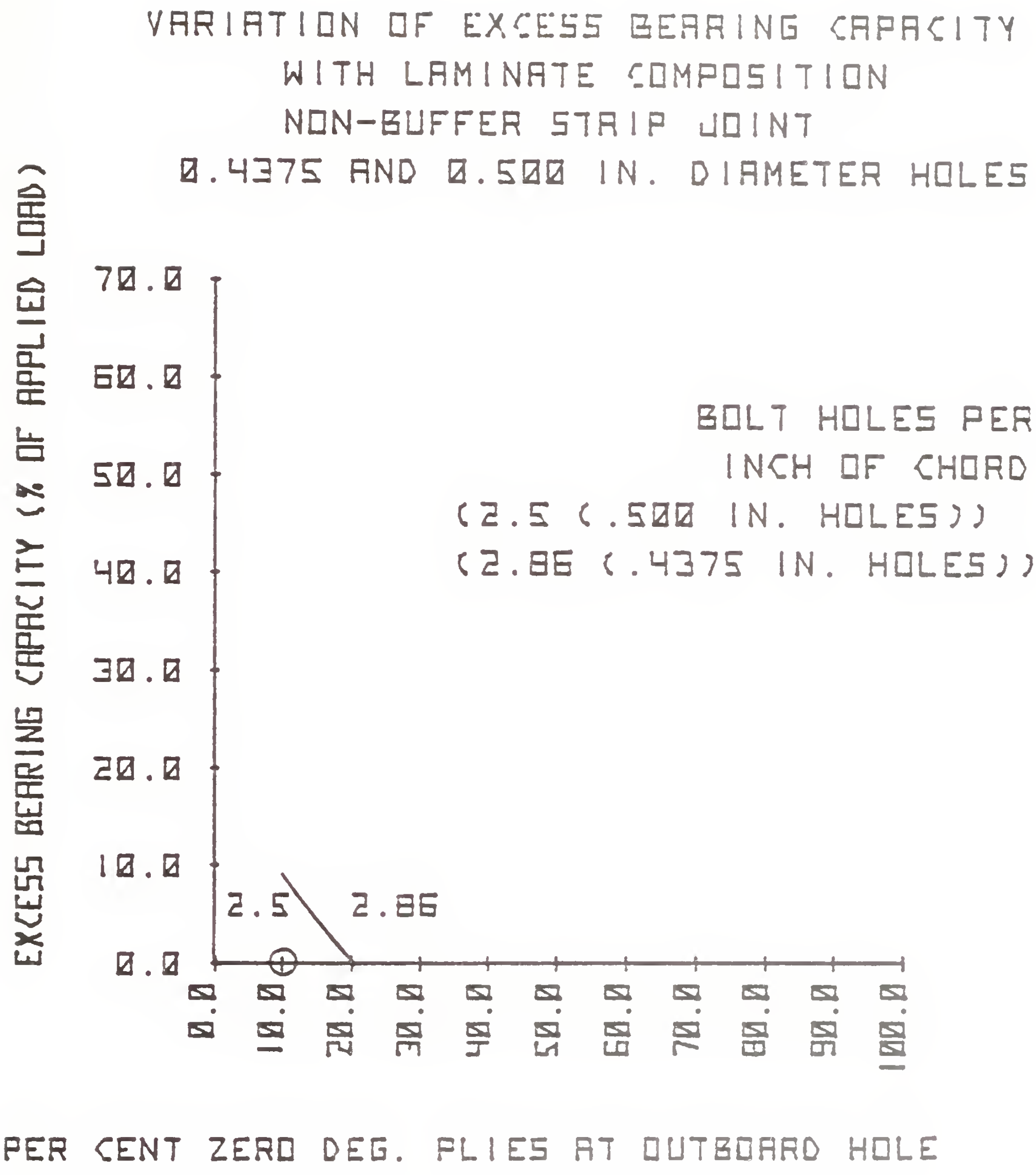


FIGURE 33. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.4375 AND 0.5 IN. DIAMETER BOLT HOLES

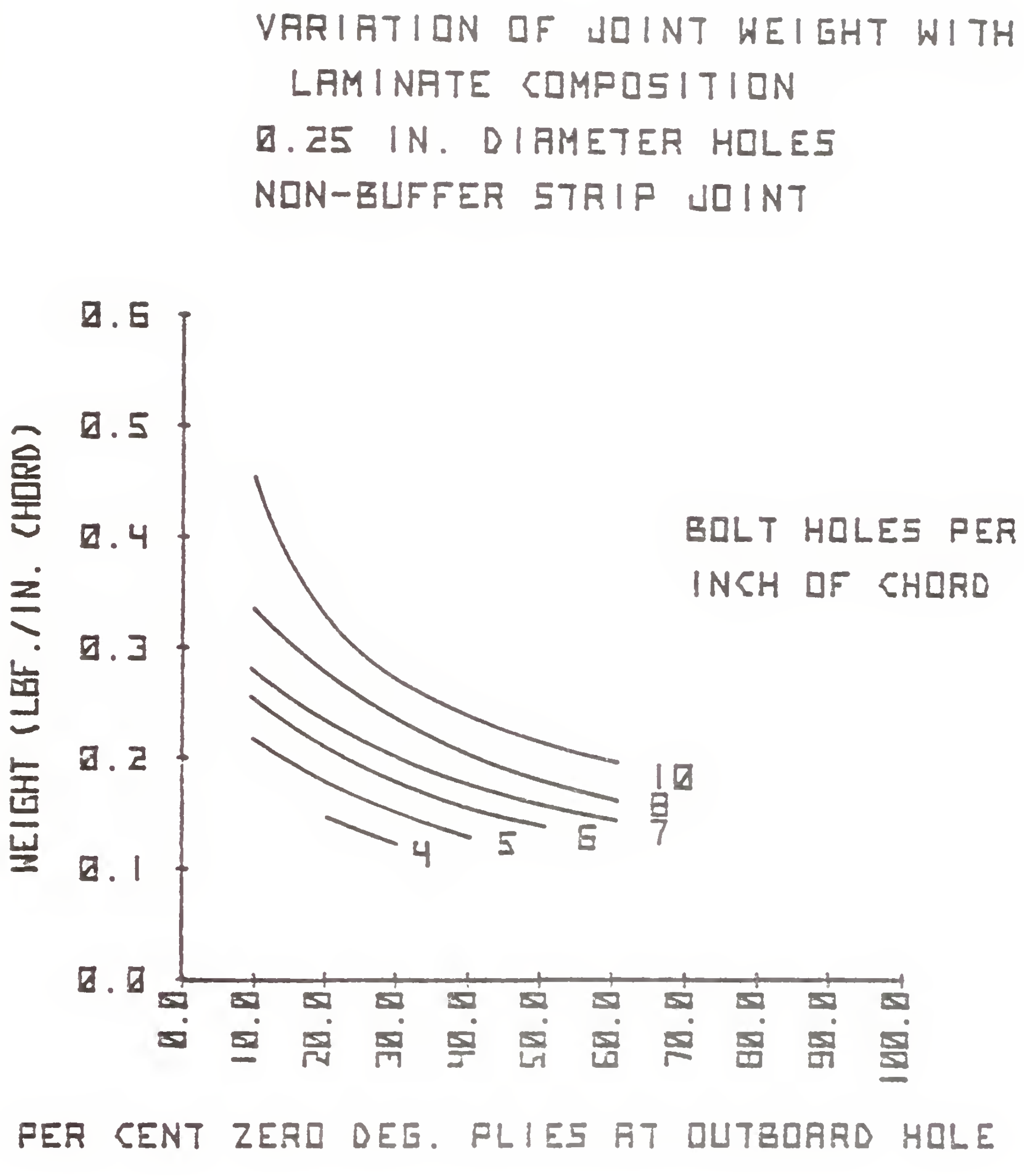


FIGURE 34. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

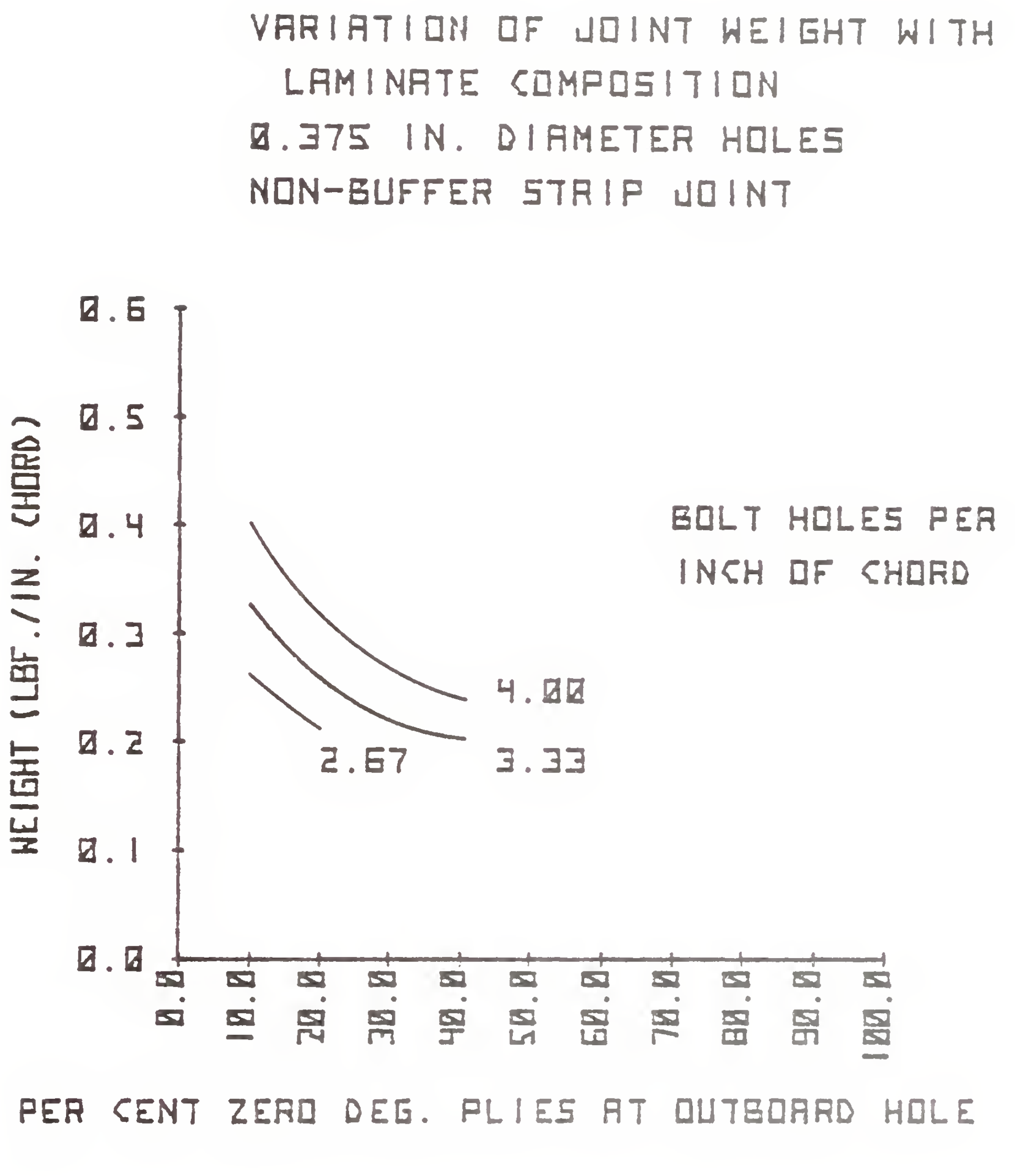


FIGURE 35. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES

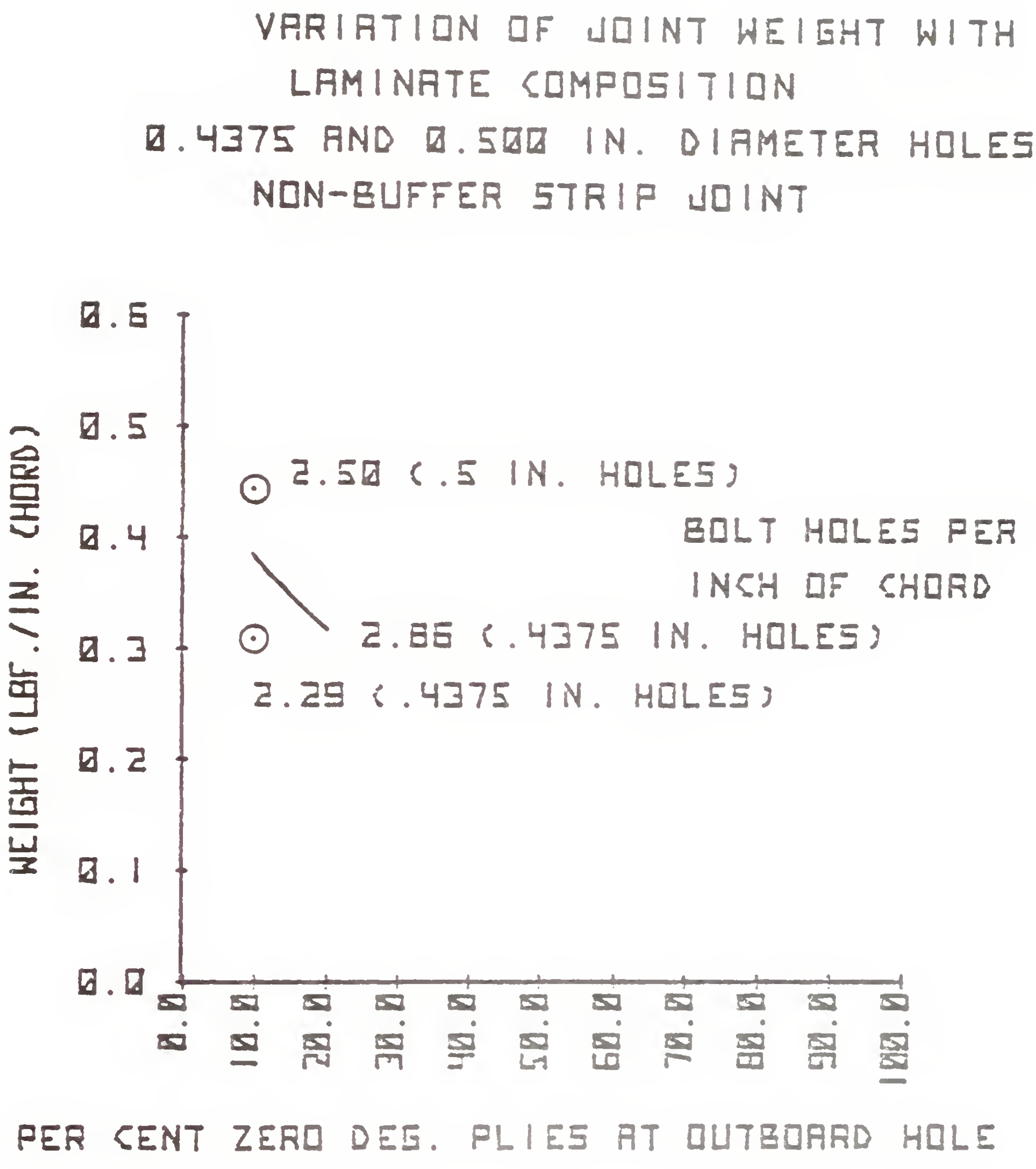


FIGURE 36. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.4375 AND 0.5 IN. DIAMETER BOLT HOLES

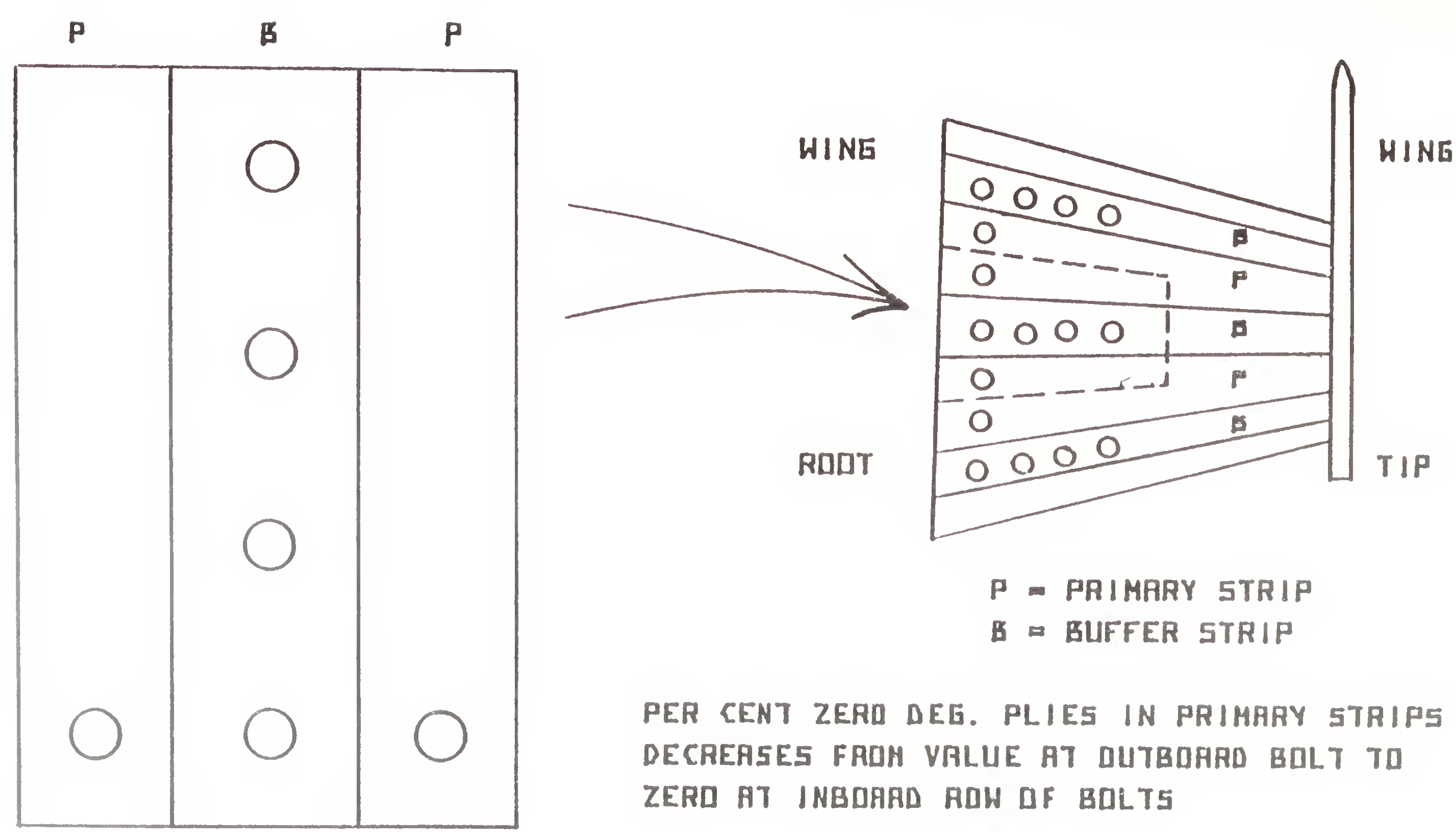
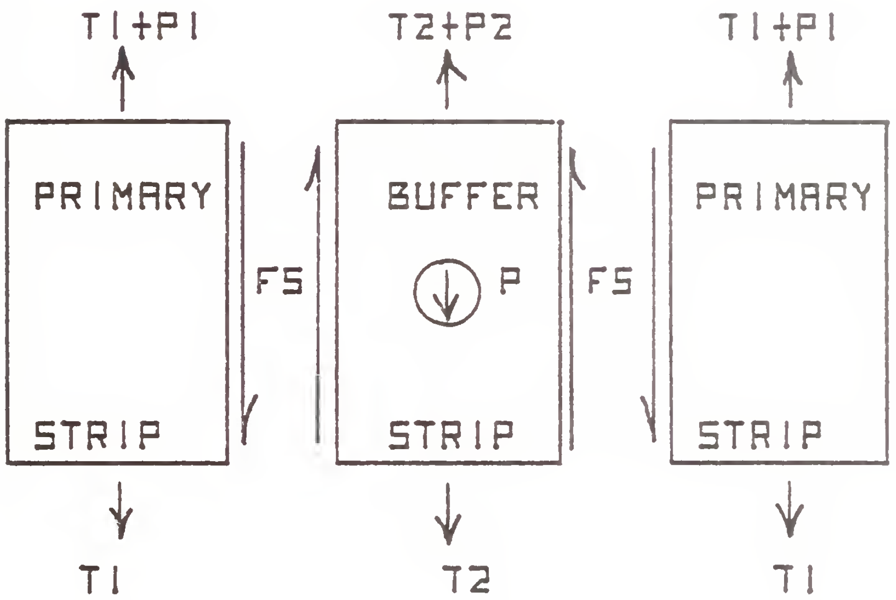


FIGURE 37. SCHEMATIC OF A WING WITH A BUFFER STRIP JOINT

MECHANISM BY WHICH BOLT LOADS ARE
REACTED IN A BUFFER STRIP JOINT



T1 = TENSILE LOAD IN PRIMARY STRIP

T2 = TENSILE LOAD IN BUFFER STRIP

P = BOLT LOAD APPLIED AT HOLE

P1 = BOLT LOAD REACTED IN PRIMARY STRIP

P2 = BOLT LOAD REACTED IN BUFFER STRIP

FS = SHEAR PASSING P1 TO PRIMARY STRIP

FIGURE 38. MECHANISM BY WHICH BOLT LOADS ARE REACTED IN A BUFFER STRIP JOINT

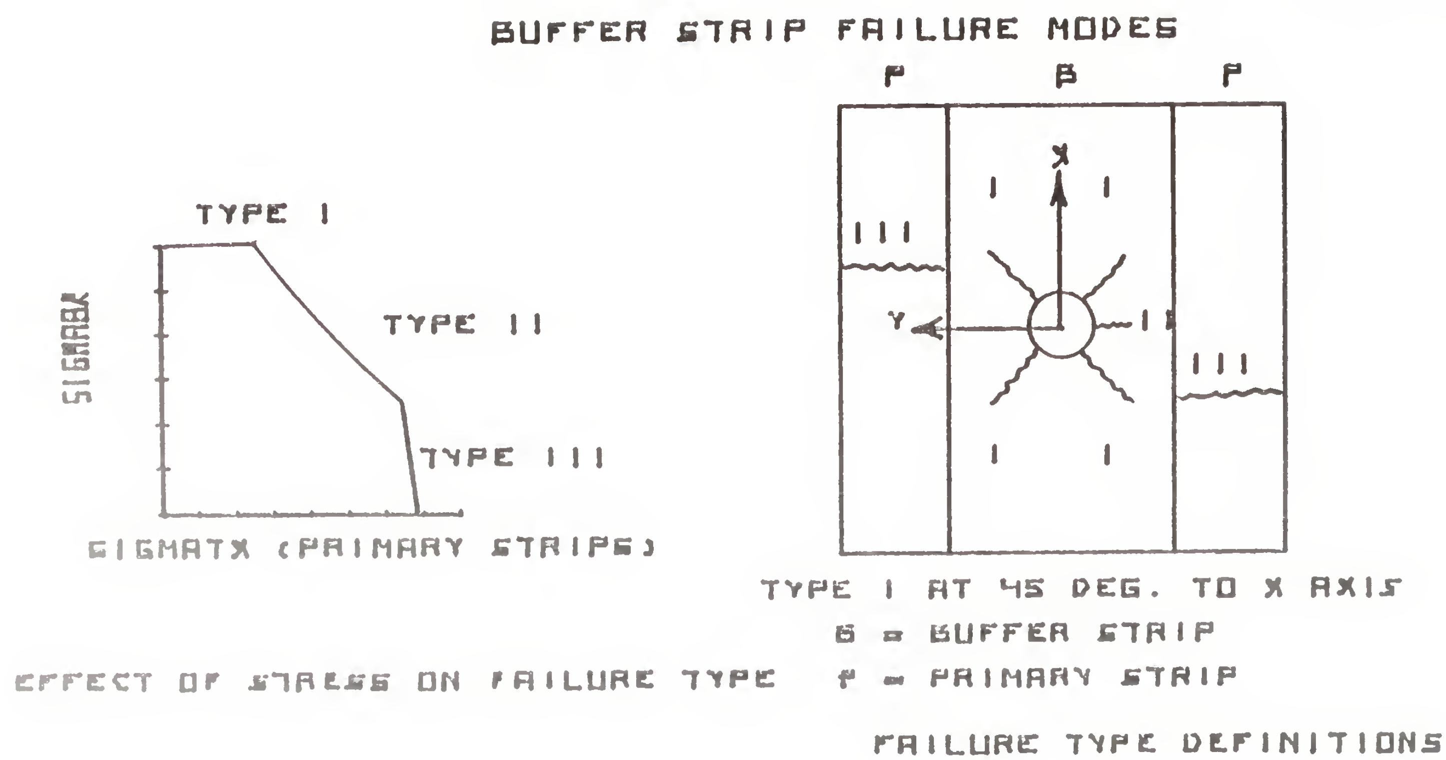


FIGURE 39. DESCRIPTION OF THE EXPECTED BUFFER STRIP FAILURE MODES

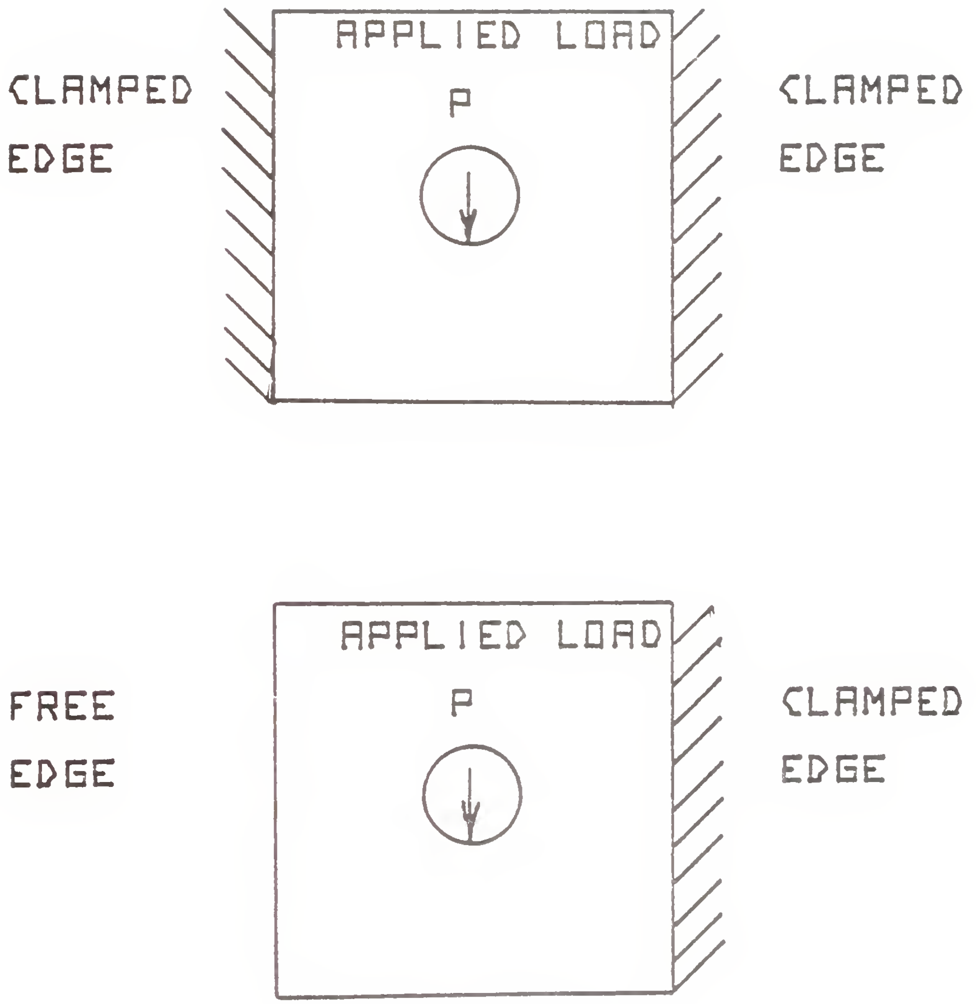


FIGURE 40. SCHEMATIC OF SHEAR LOADING TEST SPECIMENS

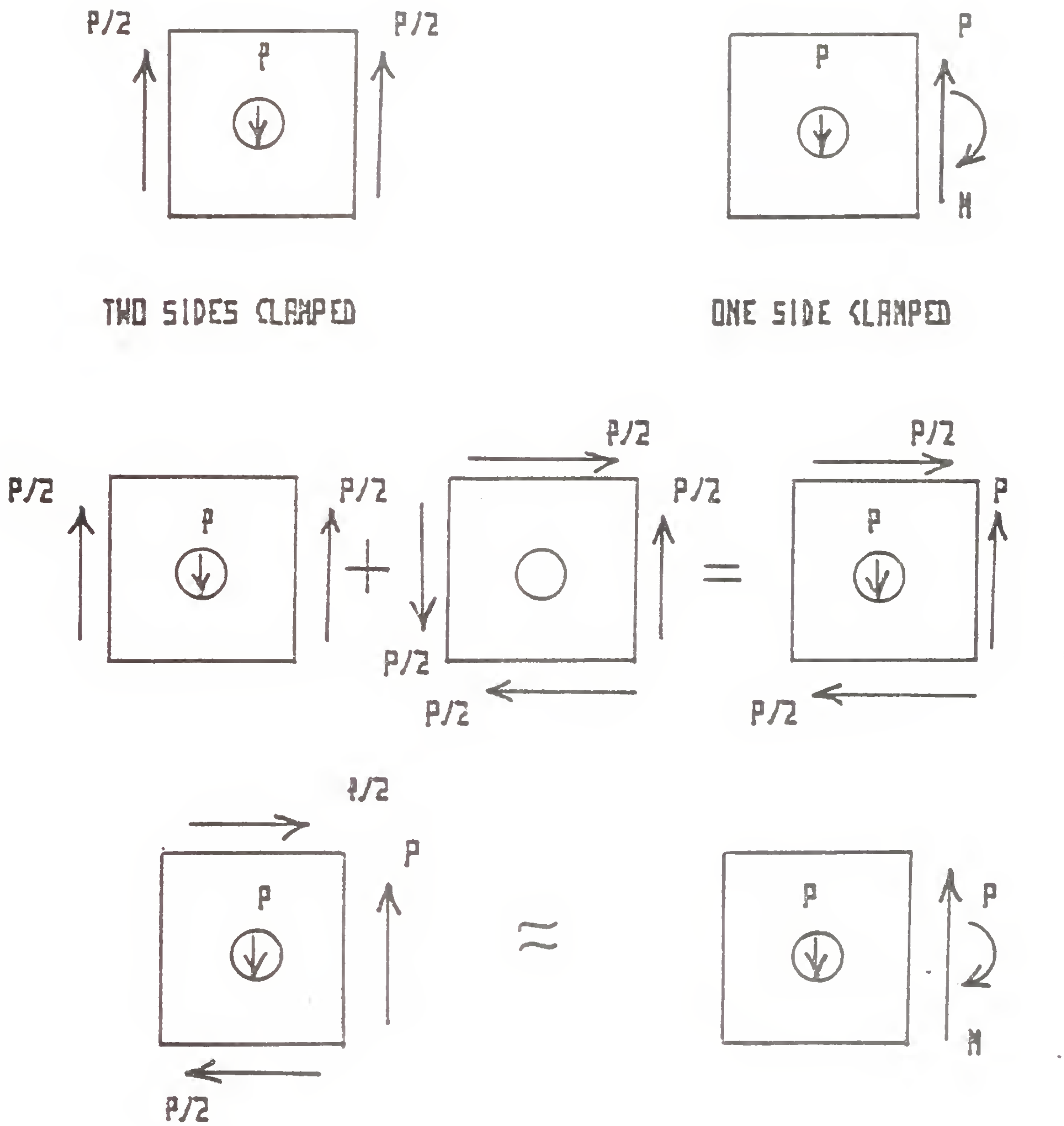


FIGURE 41. SCHEMATIC OF THE SUPERPOSITION USED TO DETERMINE SHEAR EFFECTS ON A BUFFER STRIP WITH A CENTRAL HOLE

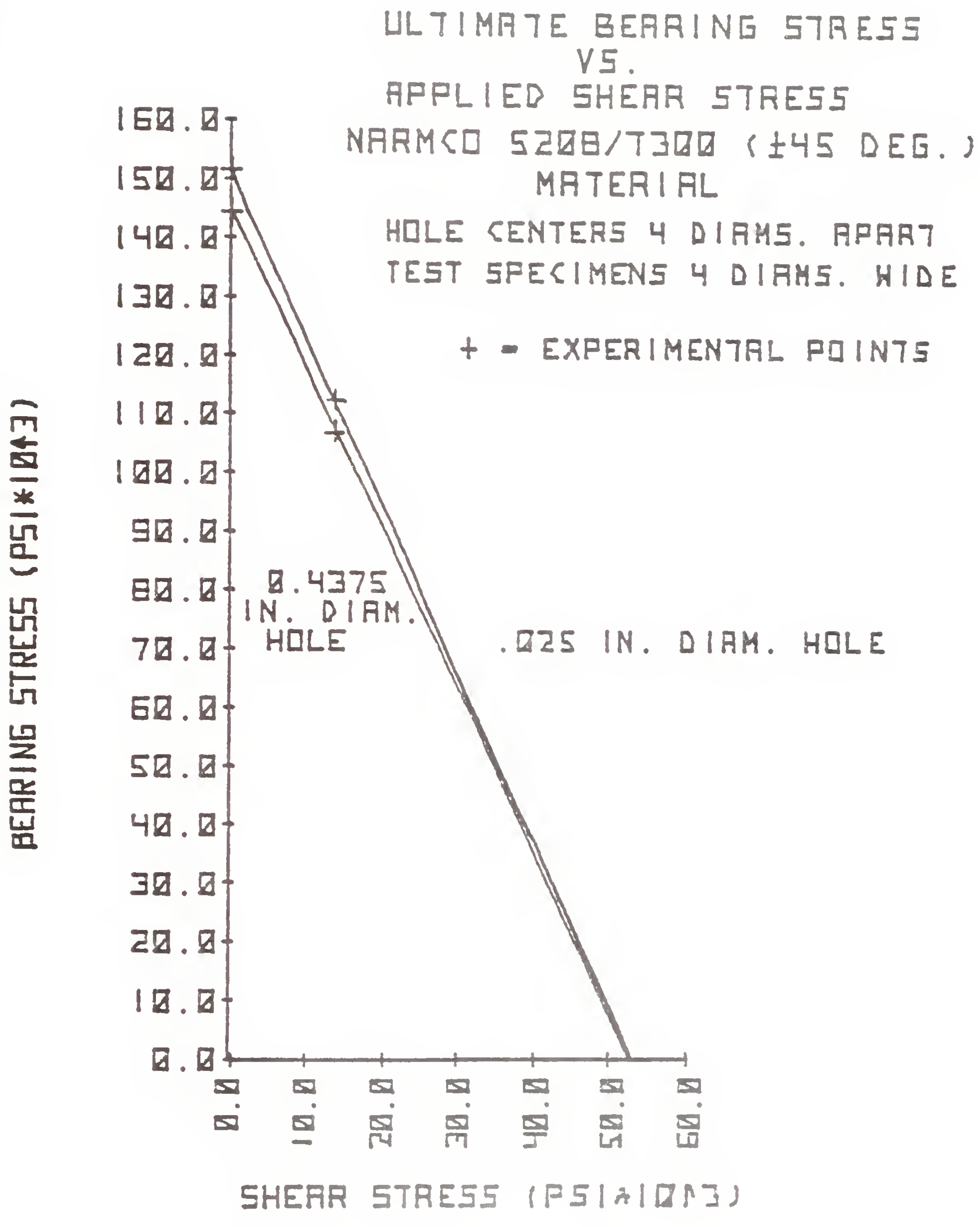


FIGURE 42. ULTIMATE BEARING STRESS-SHEAR STRESS INTERACTION CURVE FOR A FOUR HOLE DIAMETER SQUARE PLATE OF NARMCO 5208/T300 [±45] MATERIAL WITH A CENTRAL HOLE

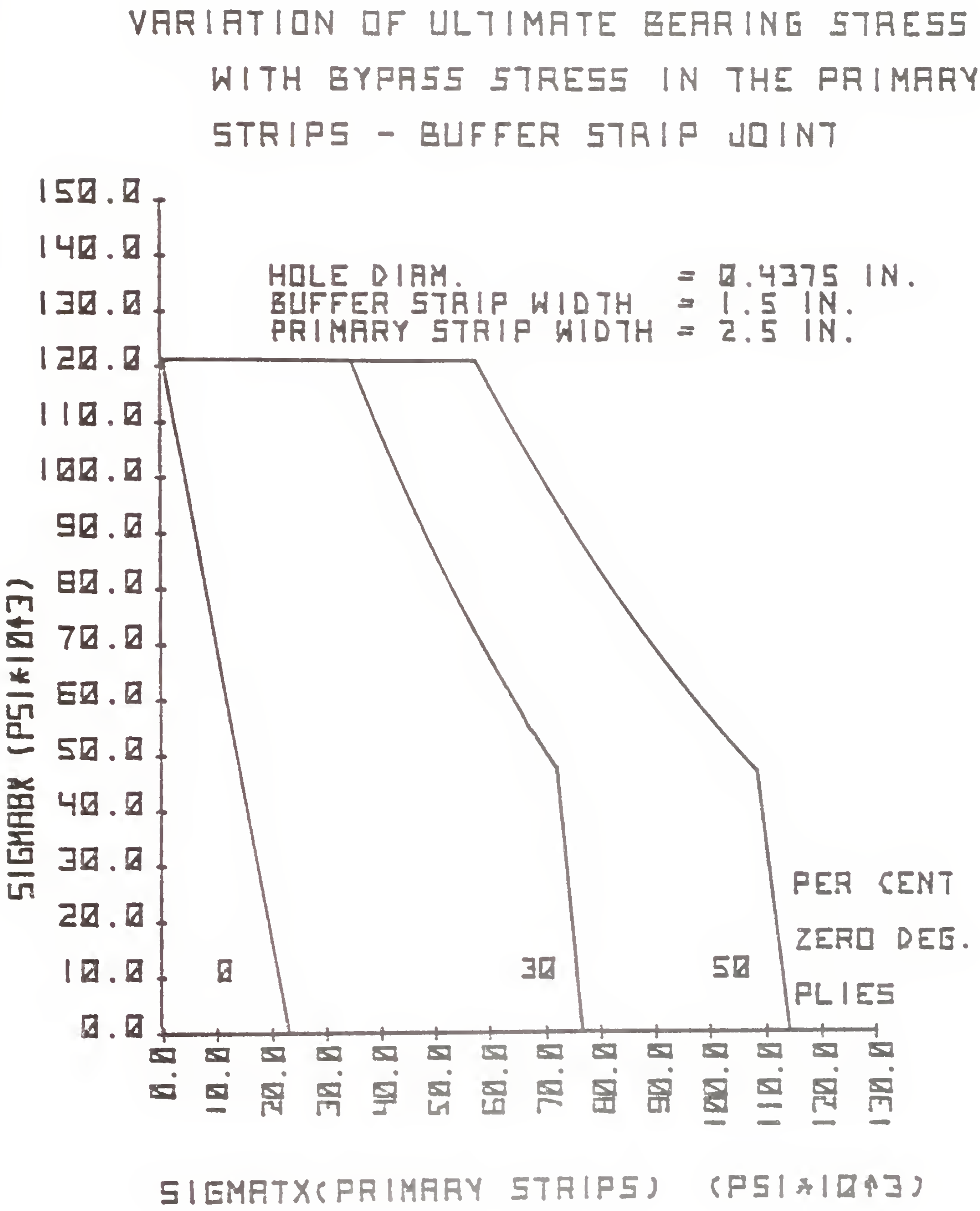


FIGURE 43. ULTIMATE STRESS INTERACTION CURVE FOR A BUFFER STRIP JOINT MADE FROM NARMCO 5208/T300 [0/±45] MATERIAL WITH 2.5 IN. WIDE PRIMARY STRIPS, A 1.5 IN. WIDE BUFFER STRIP, AND A 0.4375 IN. DIAMETER CENTRAL HOLE

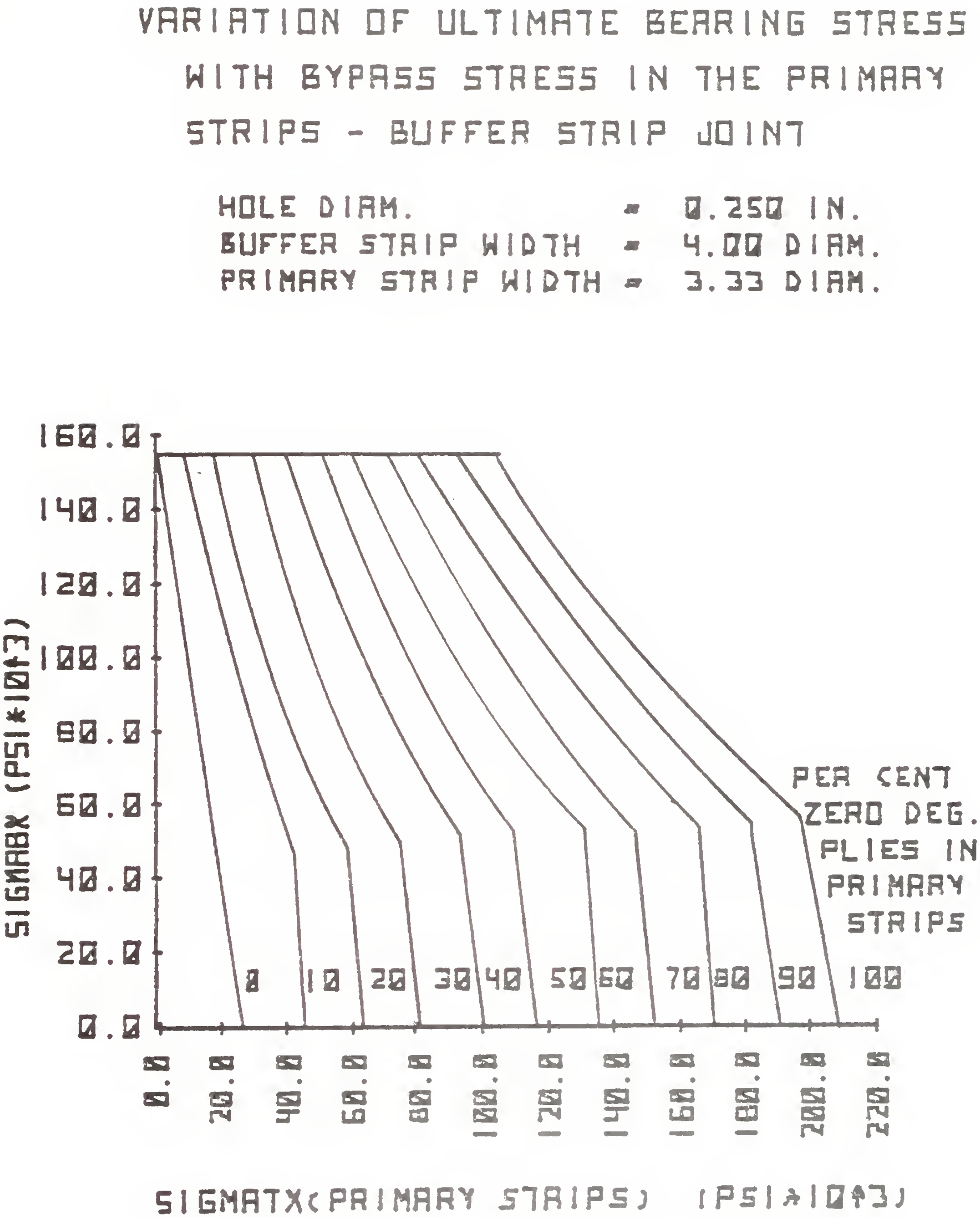


FIGURE 44. ULTIMATE STRESS INTERACTION CURVE FOR A 1.0 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO 5208/T300 [0/±45] MATERIAL WITH 0.833 IN. WIDE PRIMARY STRIPS, A 1.0 IN. WIDE BUFFER STRIP, AND A 0.25 IN. DIAMETER CENTRAL HOLE

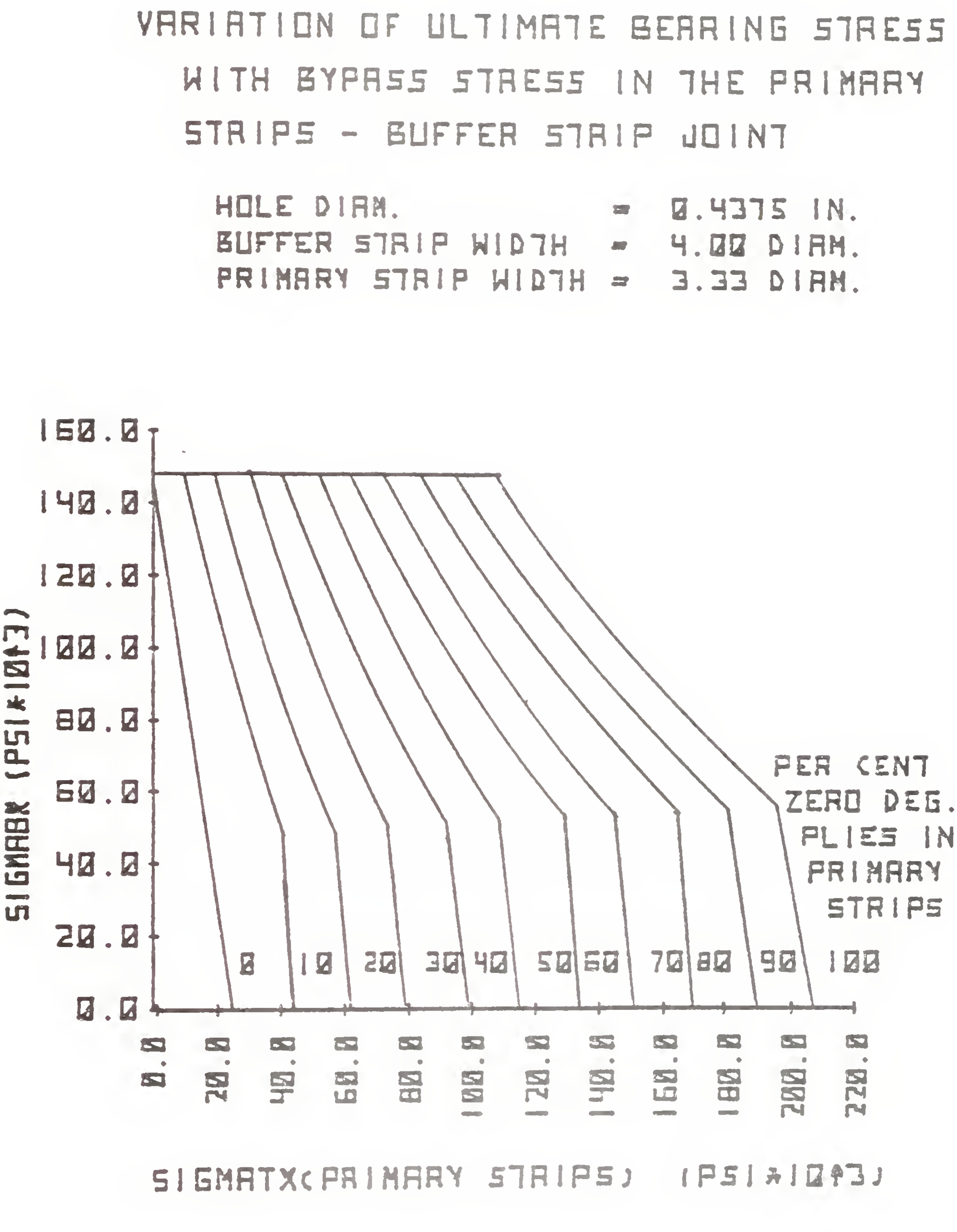


FIGURE 45. ULTIMATE STRESS INTERACTION CURVE FOR A 1.75 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO 5208/T300 $[0/\pm 45]$ MATERIAL WITH 1.46 IN. WIDE PRIMARY STRIPS, A 1.75 IN. WIDE BUFFER STRIP, AND A 0.4375 IN. DIAMETER CENTRAL HOLE

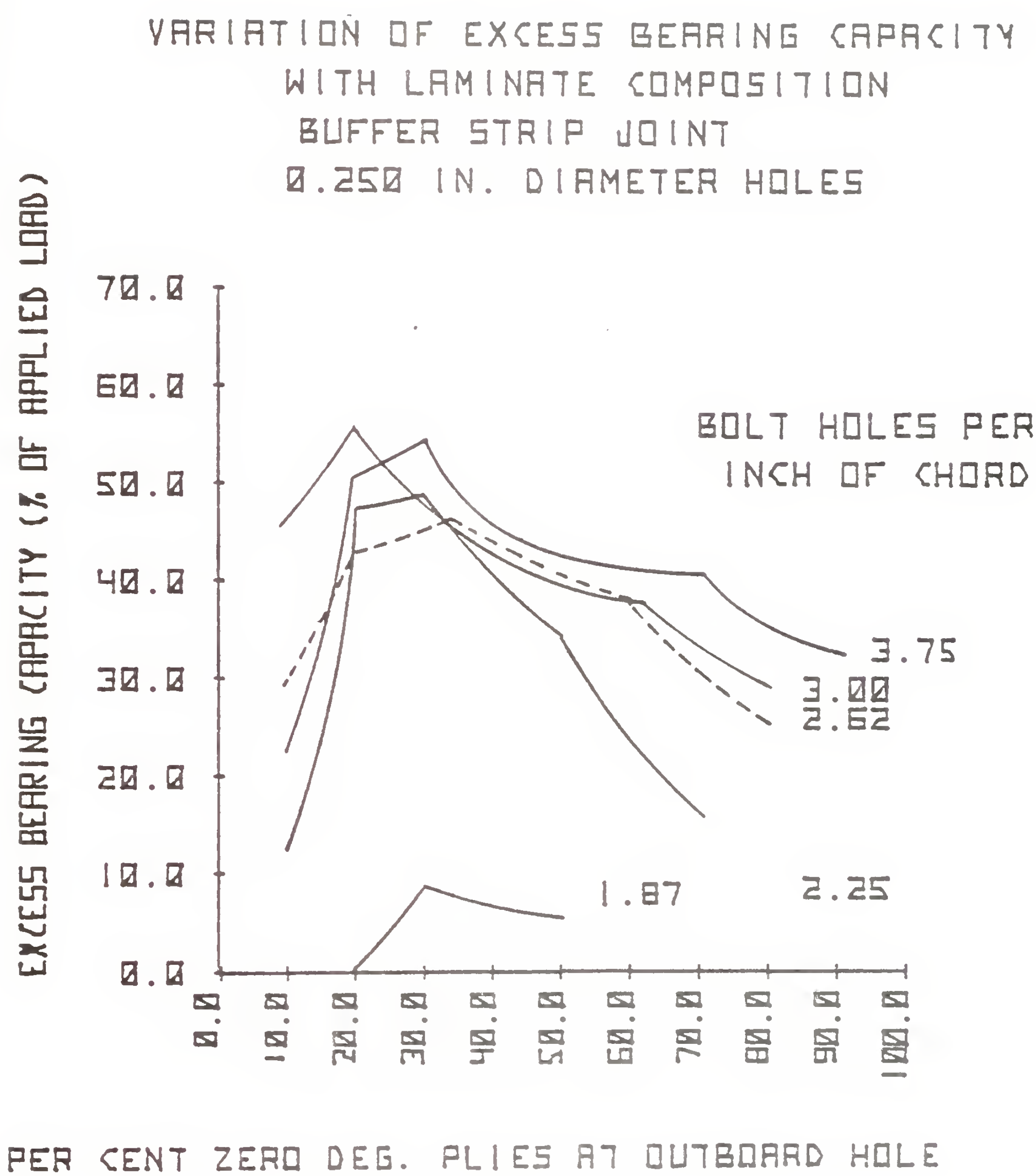


FIGURE 46. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES

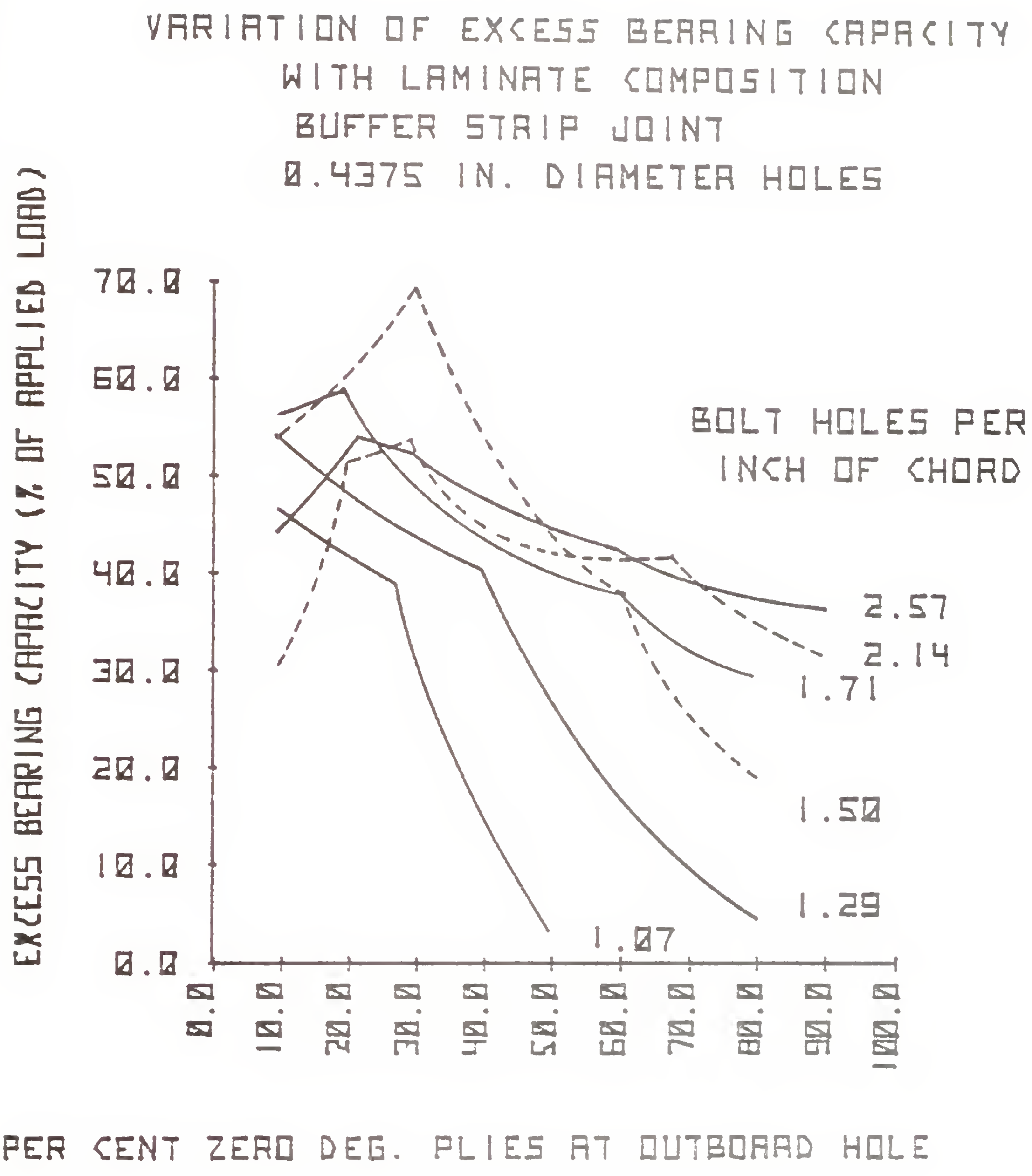


FIGURE 47. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

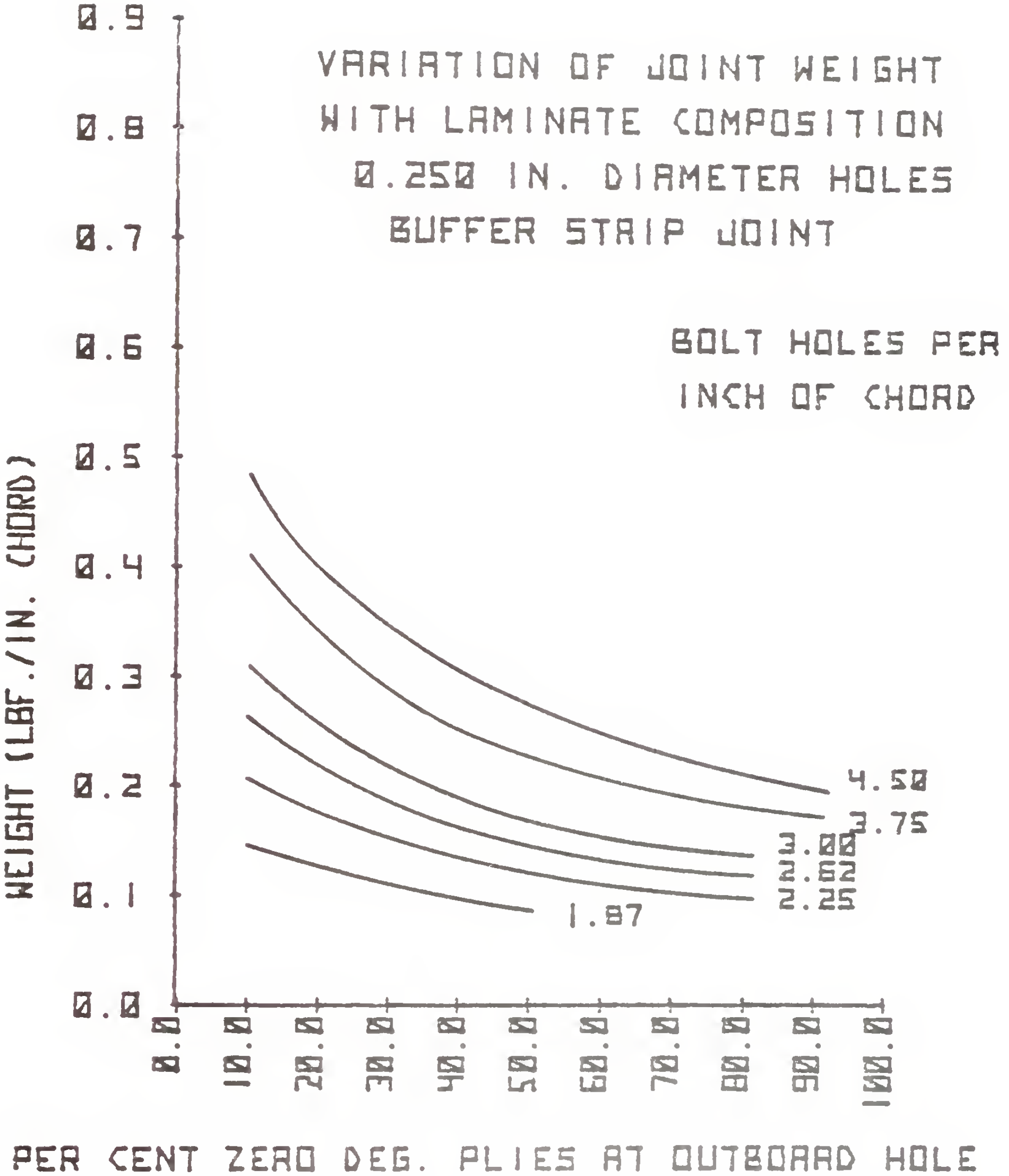


FIGURE 48. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES

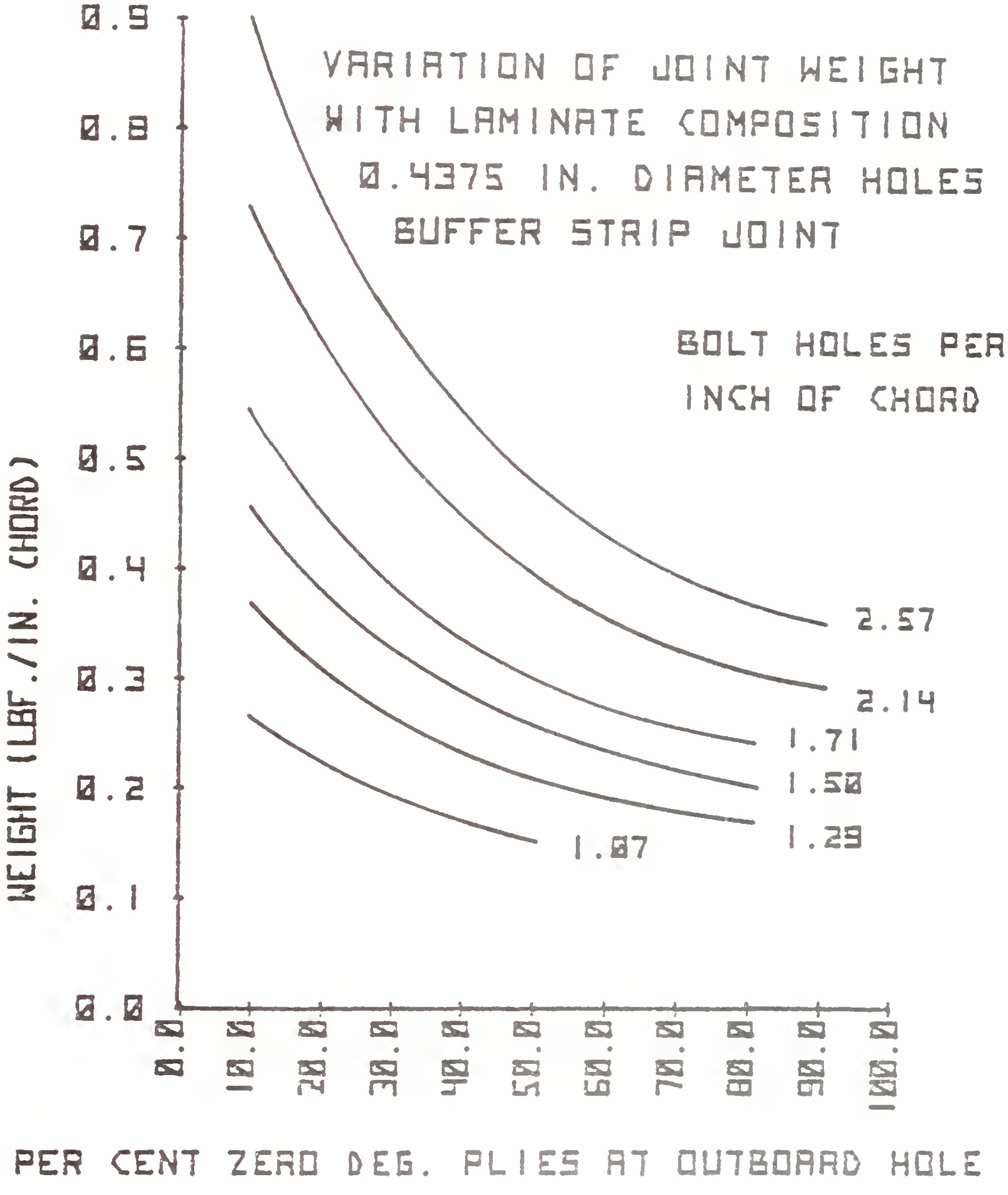


FIGURE 49. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

TABLE I

SUMMARY OF DESIGN CONDITIONS AND ASSUMPTIONS

1. The joints are made from NARMCO 5208/T300 $[0/\pm45]$ graphite-epoxy laminated material.
2. Skin thickness varies linearly within a joint.
3. All bolt holes in a joint are of the same diameter.
4. The interbolt strain level is 3000 micro-in./in.
5. In the theoretical developments for both buffer strip and non-buffer strip joints it was assumed that only tensile and shear loads were to be carried.
6. Each row of bolts reacts an equal portion of the applied tensile load.
7. The applied shear load is reacted by the inboard row of bolts.
8. The minimum number of rows of bolts in any joint is three.
9. Wing taper is disregarded.
10. The inboard row of bolts is in all ±45 degree laminate.
11. The maximum joint length is ten inches.
12. In the non-buffer strip joints, there is a four-hole-diameter spacing between adjacent rows of bolt hole centers.
13. In the non-buffer strip joint, the laminate between the inboard and next to inboard bolt holes must contain at least five per cent zero degree plies.
14. In the buffer strip joints, the buffer strip width is

four hole diameters. The primary strips are each 3.335 hole diameters wide.

15. Weight and excess bearing capacity calculations were made for assumed load conditions N_x = 20,000 lbf./in. and N_{xy} = 0.

APPENDIX A

```
C PROGRAM ISANIS                                                          MAIN
C    ************************************************************************MAIN
C     THIS PROGRAM ANALYZES THE STATE OF STRESS AND STRAIN IN A TWO       MAIN
C     DIMENSIONAL BODY COMPOSED OF EITHER ISOTROPIC OR                    MAIN
C     ORTHOTROPIC MATERIALS.THE ANALYSIS IS PERFORMED BY THE FINITE       MAIN
C     ELEMENT METHOD USING ISOPARAMETRIC ELEMENTS.                        MAIN
C     ISANIS IS A MODIFICATION OF PLISOP CODED  BY PROFESSOR              MAIN
C     GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL IN 1972.             MAIN
C     ISANIS EXPANDS PLISOP TO DEAL WITH ORTHOTROPIC ELEMENTS.            MAIN
C     CODED BY JAMES M. GILL, APRIL 1975                                  MAIN
C    ************************************************************************MAIN
C                                                                         MAIN
C    INPUT CARDS NEEDED                                                   MAIN
C                                                                         MAIN
C     TITLE CARD               (20A4)              ONE CARD PER PROBLEM   MAIN
C                                                                         MAIN
C     PROBLEM PARAMETERS       (10I5,5X,I5)        ONE CARD PER PROBLEM   MAIN
C                                                                         MAIN
C      COLUMN               VARIABLE                                      MAIN
C        1-5                NEL    NUMBER OF ELEMENTS. NEL MAX. IS 182    MAIN
C        6-10               NJT    NUMBER OF JOINTS. (MAX. IS 217)        MAIN
C       11-15               NMAT   NUMBER OF MATERIALS. (MAX IS 10)       MAIN
C       16-20               NCLOAD  NUMBER OF CONCENTRATED LOADS          MAIN
C       21-25               NPBC   NUMBER OF JOINTS WITH BOUNDARY CONDITIONS MAIN
C       26-30               NSTRES  ISOTROPIC MATERIAL PROBLEM TYPE. NSTRES MAIN
C                                    = 0 MEANS ISOTROPIC PLANE STRESS.NSTRESMAIN
C                                   = 1 MEANS ISOTROPIC PLANE STRAIN.     MAIN
C                                   FOR PLANE STRAIN, THICKNESS           MAIN
C                                   WILL BE SET EQUAL TO 1.0              MAIN
C       31-35               NGP    NUMBER OF GAUSS POINTS TO BE USED IN THE  MAIN
C                                  INTEGRATION. (MAX. IS 5 )              MAIN
C       36-40               NTLD   ISOTROPIC THERMAL LOAD INDICATOR. NTLD =1 MAIN
C                                  MEANS THERE IS AN ISOTROPIC THERMAL LOAD. MAIN
C                                  NTLD=0 MEANS NO ISOTROPIC THERMAL LOADS.  MAIN
C       41-45               NTBY   BODY FORCE LOAD INDICATOR. NTBY = 1 MEANS MAIN
C                                  A BODY FORCE LOAD IS PRESENT.NTBY= 0   MAIN
C                                  MEANS NO BODY FORCE LOADS ARE PRESENT. MAIN
C       46-50               NTIN   INITIAL STRETCH INDICATOR. NTIN = 1 MEANS MAIN
C                                  INITIAL STRETCH IS TO BE CONSIDERED.   MAIN
C                                  NTIN=0 MEANS NO INITIAL STRETCH.       MAIN
C             IF (NTLD+NTBY+NTIN) IS NOT EQUAL TO ZERO, NJT CARDS WILL BEMAIN
C             NEEDED TO INPUT THE CONSISTENT LOAD VECTOR RESULTING FROM   MAIN
C             THESE CONDITIONS. PROGRAM ISLOAD CAN BE USED TO GENERATE    MAIN
C             THIS VECTOR.                                                MAIN
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C       56-60                  NLEL  NUMBER OF LAMINATE TYPES WHOSE MATERIAL    MAIN 450
C                              PROPERTIES ARE TO  BE CALCULATED FROM            MAIN 460
C                              INPUT LAMINA PROPERTIES. (MAX. 50)               MAIN 470
C                                                                               MAIN 480
C    JOINT CARDS        (I5,5X,4G15.8,I5)             NJT CARDS PER PROBLEM     MAIN 490
C     COLUMN                      VARIABLE                                      MAIN 500
C       1-5                       JOINT NUMBER                                  MAIN 510
C      11-25                      X COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 520
C                                 RADIUS VECTOR LENGTH (POLAR COORD.)           MAIN 530
C      26-40                      Y-COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 540
C                                 POLAR ANGLE IN DEGREES (POLAR COORD.)         MAIN 550
C      41-55                      X-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 560
C      56-70                      Y-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 570
C      70-75                      COORDINATE SYSTEM INDICATOR. IND. = ZERO IFMAIN 580
C                                 THE COORDINATE SYSTEM IS CARTESIAN, IF IND MAIN 590
C                                 IS NOT ZERO, THE COORDINATE SYSTEM IS POLARMAIN 600
C                                                                               MAIN 610
C     MATERIAL PROPERTIES CARDS(2I5,7F10.2)        NMAT CARDS PER PROBLEM       MAIN 620
C      CCLUMN                VARIABLE                                           MAIN 630
C       1-05                 MATERIAL NUMBER                                    MAIN 640
C       6-10                 INDK EQ.0 (ISOTROPIC), EQ.1 (ORTHOTROPIC)          MAIN 650
C      11-20                 YOUNG'S MODULUS (ISOTROPIC MATERIAL)               MAIN 660
C                            E(L) (ORTHOTROPIC MATERIAL)                        MAIN 670
C      21-30                 POISSON'S RATIO (ISOTROPIC MATERIAL)               MAIN 680
C                            E(T) (ORTHOTROPIC MATERIAL)                        MAIN 690
C      31-40                 NOT USED (ISOTROPIC MATERIAL)                      MAIN 700
C                            NU(LT) (ORTHOTROPIC MATERIAL)                      MAIN 710
C      41-50                 NOT USED (ISOTROPIC MATERIAL)                      MAIN 720
C                            NU(TL) (ORTHOTROPIC MATERIAL)                      MAIN 730
C                   FOR ORTHOTROPIC MATERIALS, ONLY 3 OF THE ABOVE 4 ENGIN.     MAIN 740
C                   CONSTANTS (E(L),E(T),NU(LT),NU(TL)) NEED BE KNOWN. FOR ANMAIN 750
C                   UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.      MAIN 760
C      51-60                 NOT USED (ISOTROPIC MATERIALS)                     MAIN 770
C                            G(LT) (ORTHOTROPIC MATERIALS)                      MAIN 780
C      61-70                 ALPHA   COEFF. OF ISOTROPIC THERMAL EXPANSION      MAIN 790
C                            NOT USED FOR THERMALLY ANISOTROPIC MATERIALS       MAIN 800
C      71-80                 MATERIAL THICKNESS (USED FOR ELEMENTS              MAIN 810
C                            WHOSE MATERIAL PROPERTIES ARE NOT TO BE            MAIN 820
C                            CALCULATED FROM INPUT LAMINA PROPERTIES.)          MAIN 830
C                                                                               MAIN 840
C                                                                               MAIN 850
C     ELEMENT CARDS             (15I4,F10.5,I5)     NEL CARDS PER PROBLEM       MAIN 860
C       1-4                   ELEMENT IDENTIFICATION NUMBER                     MAIN 870
C       5-52                  ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)          MAIN 880
C      53-56                 ELEMENT KIND - USE                                 MAIN 890
C                             1  FOR A  FOUR  NODAL POINT ELEMENT               MAIN 900
C                             2  FOR AN EIGHT NODAL POINT ELEMENT               MAIN 910
C                             3  FOR A TWELVE NODAL POINT ELEMENT               MAIN 920
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C     57-60                MATERIAL / LAMINATE IDENTIFICATION NUMBER         MAIN 930
C     61-70                THETA (DEGREES) USED FOR ORTHOTROPIC MATERIALS    MAIN 940
C                         THETA IS THE ANGLE FROM THE PROBLEM BASED  X AXISMAIN 950
C                          TO  THE MATERIAL BASED L AXIS                     MAIN 960
C     71-75                ENTER 1 FOR A LAMINATED ELEMENT WHOSE PROPERTIESMAIN 970
C                          ARE TO BE CALCULATED FROM INPUT LAMINA            MAIN 980
C                          PROPERTIES.                                       MAIN 990
C                                                                            MAIN1000
C  BOUNDARY CONDITION CARDS  (2I5)          NPBC  CARDS PER PROBLEM         MAIN1010
C   COL.   VARIABLE                                                          MAIN1020
C    1-5  JOINT NUMBER                                                       MAIN1030
C   6-10  CODE   1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO MAIN1040
C                2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO MAIN1050
C                3 FOR A FIXED JOINT, BOTH DISP. COMPONENTS ARE ZERO     MAIN1060
C                4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT. MAIN1070
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 41-50          MAIN1080
C                5 FOR A ROLLER ON A Y AXIS, THE DISP. IN THE X DIRECT.  MAIN1090
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 31-40          MAIN1100
C                6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y      MAIN1110
C                  DIRECT. ON THE JOINT CARD IN COL.31-40 AND COL.41-50  MAIN1120
C                7 FOR A ROLLER ON AN AXIS INCLINED ALPHA DEGREES        MAIN1130
C                  WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY        MAIN1140
C                  CONTAIN A TANGENTIAL LOAD IN COL. 31-40 AND THE       MAIN1150
C                  ANGLE ALPHA DEGREES IN COL. 41-50                     MAIN1160
C                                                                            MAIN1170
C   LAMINATE CARD DECK        AT LEAST TWO CARDS PER LAMINATE TYPE         MAIN1180
C   CARD ONE          2(I5)                                                  MAIN1190
C     COLUMN          VARIABLE                                               MAIN1200
C       1-5           LAMTP        LAMINATE TYPE NUMBER(MAX.50)             MAIN1210
C      6-10           LYRNO        NO. LAMINAE IN THIS LAMTP (MAX.100)      MAIN1220
C   CARD TWO         4(I3,F8.3,F9.4)                                         MAIN1230
C     COLUMN          VARIABLE                                               MAIN1240
C       1-3           MATERIAL NUMBER                                        MAIN1250
C      4-11           LAMINA THICKNESS                                       MAIN1260
C     12-20           LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1270
C     21-23           MATERIAL NUMBER                                        MAIN1280
C     24-31           LAMINA THICKNESS                                       MAIN1290
C     32-40           LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1300
C     41-43           MATERIAL NUMBER                                        MAIN1310
C     44-51           LAMINA THICKNESS                                       MAIN1320
C     52-60           LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1330
C     61-63           MATERIAL NUMBER                                        MAIN1340
C     64-71           LAMINA THICKNESS                                       MAIN1350
C     72-80           LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT  MAIN1360
C   AS MANY OF THESE SECOND TYPE CARDS AS NEEDED TO SPECIFY LYRNO LAYERSMAIN1370
C   MAY BE USED                                                              MAIN1380
C   REPEAT THE SEQUENCE WITH NEW CARD TYPE 1 AND 2 AS NEEDED TO            MAIN1390
C   SPECIFY ALL  NLEL LAMINATE TYPES                                         MAIN1400
```

```
C
C     CONSISTENT LOAD VECTOR (I10,2G25.16)      NJT CARDS PER PROBLEM
C      NO CARDS NEEDED IF (NTLD+NTBY+NTIN)=0.
C      COL.     VARIABLE
C       1-10  JOINT NUMBER
C      11-35  X COMPONENT OF LOAD
C      36-70  Y COMPONENT OF LOAD
C
C     AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF
C     A RUN MUST CONTAIN THE WORD     STOP     IN  THE FIRST FOUR COLUMNS
C
C           MAIN  PROGRAM
C
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      INTEGER *4LMK,LML,NBAND2
      CCMMON /SOL/ BGK(434,86),ALOAD(434),ABGN
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCCN(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCCN(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
      CCMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
      DIMENSION REACT(218,2), NCODE(217)
      EQUIVALENCE (BGK(1,1),REACT(1,1))
      MEL = 182
      MJT = 217
      MMT = 10
      MBD = 86
      MLEL = 50
      MLYR = 100
      PI = 3.14159265359D0
      ZRO = 0.0D0
    1 NBAND = 0
      NBAND2 = NBAND
C
      DO 2 I=1,MJT
    2 NCODE(I) = 0
C
      CALL INPUT
      NEQ = 2*NJT
      NPEL = NCON(1,13)*4
C
      DC 5 I=1,NEL
C
      DO 3 J=1,NPEL
    3 LM(J) = NCON(I,J)
```

```
C
      NPELM = NPEL-1
C
      DC 4 K=1,NPELM
      LMK = LM(K)
      JK = K+1
C
      DC 4 L=JK,NPEL
      LML = LM(L)
    4 NBAND2 = MAXO(NBAND2,IABS(LMK-LML))
C
    5 CONTINUE
C
      NBAND = NBAND2
      NBAND = (NBAND+1)*2
      IF (NBAND.GT.MBD) GO TO 16
      WRITE (6,17) NBAND
C
      DO 6 I=1,NEQ
      ALOAD(I) = ZRO
C
      DO 6 J=1,NBAND
    6 BGK(I,J) = ZRO
C
      CALL MEFGE (NPEL,NEC)
      CALL BCOND (NBAND)
      CALL LDLT (NEQ,NBAND)
C
      DO 7 I=1,NPBC
      IJT = NBC(I,1)
    7 NCODE(IJT) = NBC(I,2)
C
C
      DO 10 I=1,NJT
      II = I*2-1
      KODE = NCODE(I)
      IF (KODE.EQ.7) GO TO 9
      IF ((KODE.EQ.5).OR.(KODE.EQ.6)) GC TO 8
      ALOAD(II) = CLOAD(I,1)+ALOAD(II)
    8 II = II+1
      IF ((KODE.EQ.4).OR.(KODE.EQ.6)) GO TO 10
      ALOAD(II) = CLOAD(I,2)+ALOAD(II)
      GO TO 10
    9 ALF = CLOAD(I,2)*PI/180.0D0
      CCSA = DCOS(ALF)
      SINA = DSIN(ALF)
      IIP = II+1
      A1 = ALOAD(II)*COSA+ALOAD(IIP)*SINA+CLOAD(I,1)
```

```
      A2 = ALOAD(IIP)*COSA-ALOAD(II)*SINA                               MAIN2370
      ALOAD(II) = A1                                                    MAIN2380
      ALCAD(IIP) = A2                                                   MAIN2390
   10 CONTINUE                                                          MAIN2400
C                                                                       MAIN2410
      SXL = ZRO                                                         MAIN2420
      SYL = ZRO                                                         MAIN2430
C                                                                       MAIN2440
      DO 12 I=1,NJT                                                     MAIN2450
      IX = 2*I-1                                                        MAIN2460
      IY = IX+1                                                         MAIN2470
      IF (NCODE(I).EQ.7) GO TO 11                                       MAIN2480
      SXL = SXL+ALOAD(IX)                                               MAIN2490
      SYL = SYL+ALOAD(IY)                                               MAIN2500
      GO TO 12                                                          MAIN2510
   11 ALF = CLOAD(I,2)*PI/180.0D0                                       MAIN2520
      COSA = DCOS(ALF)                                                  MAIN2530
      SINA = DSIN(ALF)                                                  MAIN2540
      SXL = SXL+ALOAD(IX)*COSA-ALOAD(IY)*SINA                           MAIN2550
      SYL = SYL+ALOAD(IX)*SINA+ALOAD(IY)*COSA                           MAIN2560
   12 CCNTINUE                                                          MAIN2570
C                                                                       MAIN2580
      WRITE (6,24)                                                      MAIN2590
      WRITE (6,22) SXL,SYL                                              MAIN2600
      CALL SOLV (NEQ,NBAND)                                             MAIN2610
C                                                                       MAIN2620
      DO 13 I=1,NPBC                                                    MAIN2630
C                                                                       MAIN2640
      DC 13 J=1,2                                                       MAIN2650
   13 REACT(I,J) = ZRO                                                  MAIN2660
C                                                                       MAIN2670
      CALL DISPL                                                        MAIN2680
      WRITE (6,19)                                                      MAIN2690
C                                                                       MAIN2700
      DC 14 I=1,NJT                                                     MAIN2710
      II = I*2-1                                                        MAIN2720
      III = II+1                                                        MAIN2730
   14 WRITE (6,18) I,ALOAD(II),ALOAD(III)                               MAIN2740
C                                                                       MAIN2750
      WRITE (6,20)                                                      MAIN2760
C                                                                       MAIN2770
      DC 15 I=1,NPBC                                                    MAIN2780
   15 WRITE (6,18) NBC(I,1),REACT(I,1),REACT(I,2)                       MAIN2790
C                                                                       MAIN2800
      WRITE (6,21)                                                      MAIN2810
      MJP = MJT+1                                                       MAIN2820
      WRITE (6,22) REACT(MJP,1),REACT(MJP,2)                            MAIN2830
      CALL STRESS (NPEL)                                                MAIN2840
```

```
      GO TO 1
   16 WRITE (6,23) MBD,NBAND
      STOP
C
   17 FORMAT (1X,'PROBLEM BANDWIDTH EQUALS ',I5)
   18 FORMAT (2X,I3,2G15.4)
   19 FORMAT (//,' DISPLACEMENTS',//,3X,' JOINT',1X,'  U',12X,'  V',/)
   20 FORMAT (//,'      REACTIONS',//,3X,' JOINT',1X,' RX',12X,' RY',/)
   21 FCRMAT (/,' SUM OF THE REACTIONS AT THE SUPPORTS',/)
   22 FORMAT (5X,2G15.4)
   23 FORMAT (5X,' MAX. BAND (',I13,') EXCEEDED, NBAND = ',I14)
   24 FCRMAT (/,'  SUM OF THE EXTERNAL LOADS',/)
      END
      SUBROUTINE INPUT
C   THIS SUBROUTINE READS INPUT DATA FOR THE PROBLEM
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      CCMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      CCMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
      CCMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
      DATA CHK/'STOP    '/
      ZRO = 0.0D0
      PI = 3.14159265359D0
      READ (5,30) TITLE
      WRITE (6,31) TITLE
      IF (TITLE(1).EQ.CHK) STOP
C
      DO 1 I=1,MJT
      THNP(I) = ZRO
C
      DO 1 J=1,2
      NBC(I,J) = 0
      COARD(I,J) = ZRO
    1 CLCAD(I,J) = ZRO
C
C
      DC 2 I=1,MMT
      INDK(I) = 0
C
      DC 2 J=1,7
    2 ELCON(I,J) = ZRO
C
C
      DO 3 I=1,MEL
```

```
      ANGL(I) = ZRO
C
      DO 3 J=1,15
    3 NCON(I,J) = 0
C
C
      DO 4 I=1,MLEL
      LYRNO(I) = 0
C
      DO 4 J=1,MLYR
      LAMAT(I,J) = 0
      ORLAM(I,J) = ZRO
    4 TKLAM(I,J) = ZRO
C
C
C     READ PROBLEM PARAMETERS
C
      READ (5,32) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NLE
     1L
      IF (NEL.GT.MEL) GO TO 23
      IF (NJT.GT.MJT) GO TO 24
      IF (NMAT.GT.MMT) GO TO 25
      IF (NCLOAD.GT.MJT) GO TO 26
      IF (NPBC.GT.MJT) GO TO 27
      IF (NLEL.GT.MLEL) GO TO 28
      WRITE (6,33) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NLEL
C
C     READ COORDINATES OF JOINTS AND CONCENTRATED LOADS
C
      WRITE (6,34)
C
      DO 5 I=1,NJT
      READ (5,35) IJT,COARD(IJT,1),COARD(IJT,2),CLOAD(IJT,1),CLOAD(IJT,2
     1),IND
      IF (IND.EQ.0) GO TO 5
      ALF = COARD(IJT,2)*PI/180.0D0
      COSA = DCOS(ALF)
      SINA = DSIN(ALF)
      XC = COARD(IJT,1)*COSA
      YC = COARD(IJT,1)*SINA
      COARD(IJT,1) = XC
      CCARD(IJT,2) = YC
    5 CONTINUE
C
C
      DO 6 I=1,NJT
    6 WRITE (6,36) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)
C
```

```
C
C     READ MATERIAL PROPERTIES CARDS
C
C
      DO 7 I=1,NMAT
    7 READ (5,37) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)
C
      WRITE (6,38)
C
      DC 8 I=1,NMAT
      WRITE (6,39) I,(ELCON(I,J),J=1,7),INDK(I)
    8 CONTINUE
C
C
C     READ  THE ELEMENT CARDS
C
      WRITE (6,40)
      WRITE (6,41)
C
      DO 9 I=1,NEL
    9 READ (5,42) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)
C
C
      DO 10 I=1,NEL
   10 WRITE (6,43) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)
C
C
C     READ BOUNDARY CONDITION CARDS
C
      WRITE (6,44)
C
      DO 11 I=1,NPBC
      READ (5,45) (NBC(I,J),J=1,2)
   11 WRITE (6,46) (NBC(I,J),J=1,2)
C
C
      DC 12 I=1,NPBC
C
      DO 12 J=I,NPBC
      IF (NBC(I,1).LE.NBC(J,1)) GO TO 12
      NT1 = NBC(I,1)
      NT2 = NBC(I,2)
      NBC(I,1) = NBC(J,1)
      NBC(I,2) = NBC(J,2)
      NBC(J,1) = NT1
      NBC(J,2) = NT2
   12 CONTINUE
C
```

```
      NSV = 0
C
      DO 13 I=1,NPBC
      IF (NBC(I,2).NE.7) GO TO 13
      NSV = NSV+1
   13 CCNTINUE
C
      NPBCM = NPBC-1
      IF (NSV.EQ.0) GO TO 18
C
      DO 16 I=1,NSV
C
      DC 15 J=1,NPBCM
      IF (NBC(J,2).NE.7) GO TO 15
      NT1 = NBC(J,1)
C
      DO 14 K=J,NPBCM
      KP = K+1
      NBC(K,1) = NBC(KP,1)
   14 NBC(K,2) = NBC(KP,2)
C
      NBC(NPBC,1) = NT1
      NBC(NPBC,2) = 7
      GO TO 16
   15 CCNTINUE
C
   16 CONTINUE
C
      WRITE (6,44)
C
      DC 17 I=1,NPBC
   17 WRITE (6,46) (NBC(I,J),J=1,2)
C
   18 CCNTINUE
      IF (NLEL.EQ.0) GO TO 21
C
C     READ LAMINATE CARD DECK
C
C
      DO 19 I=1,NLEL
      READ (5,47) LAMTP,LYRNO(LAMTP)
      L = LYRNO(LAMTP)
      IF (L.GT.MLYR) GO TO 29
      READ (5,48) ((LAMAT(LAMTP,J),TKLAM(LAMTP,J),ORLAM(LAMTP,J)),J=1,L)
   19 CONTINUE
C
      WRITE (6,49)
C
```

```
      DC 20 I=1,NLEL
      WRITE (6,50) I,LYRNO(I)
      L = LYRNO(I)
C
      DO 20 J=1,L
   20 WRITE (6,51) J,LAMAT(I,J),TKLAM(I,J),ORLAM(I,J)
C
C
C     REAC THE CONSISTENT LOAD VECTOR (IF REQUIRED)
C
   21 IF ((NTLD+NTBY+NTIN).EQ.0) RETURN
C
      DO 22 I=1,NJT
      READ (5,52) IJT,CLX,CLY,THNP(I)
      CLOAD(IJT,1) = CLOAD(IJT,1)+CLX
      CLOAD(IJT,2) = CLOAD(IJT,2)+CLY
   22 CCNTINUE
C
      RETURN
   23 WRITE (6,53) MEL,NEL
      STOP
   24 WRITE (6,54) MJT,NJT
      STOP
   25 WRITE (6,55) MMT,NMAT
      STOP
   26 WRITE (6,56) MJT,NCLOAD
      STOP
   27 WRITE (6,57) MJT,NPBC
      STOP
   28 WRITE (6,58) MLEL,NLEL
      STOP
   29 WRITE (6,59) MLYR,L
      STOP
C
   30 FCRMAT (10A8)
   31 FCRMAT (1X,10A8)
   32 FCRMAT (10I5,5X,I5)
   33 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TCTAL NUMBER OF JOINTS='
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOAD
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' TH
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS PCINTS USED ',I5,//,'
     4NUMBER OF LAMINATE TYPES USED',I5)
   34 FCRMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7X
     1,'X LOAD',9X,' Y LOAD'//)
   35 FORMAT (I5,5X,4G15.8,I5)
   36 FCRMAT (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)
   37 FCRMAT (2I5,7F10.2)
   38 FORMAT (///,' MATERIAL PROPERTIES',//,1X,'MATERIAL    ',2X,'E,E(L)
```

```
  1',7X,'NU,E(LT)',5X,'NU(LT)',8X,'NU(TL)',10X,'G(LT)',12X,'ALPHA',9XINPT2280
  2,'THICKNESS',9X,'INDK')                                               INPT2290
39 FORMAT (' ',1X,I3,5X,7(2X,G13.6),2X,I5)                                INPT2300
40 FORMAT (///,' ELEMENT CONNECTIVITY AND MATERIAL CARDS')                INPT2310
41 FORMAT (//,' ELEMENT NUMBER',20X,'JOINT NUMBERS',30X,'KIND',3X,'MAINPT2320
  1TERIAL',3X,'THETA(DEG.)',3X,'LAM. IND.')                              INPT2330
42 FORMAT (15I4,F10.5,I5)                                                 INPT2340
43 FORMAT (6X,I3,6X,12I5,1X,I5,5X,I5,1X,F10.5,5X,I5)                      INPT2350
44 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)INPT2360
45 FORMAT (2I5)                                                           INPT2370
46 FORMAT (5X,I5,I10)                                                     INPT2380
47 FORMAT (2I5)                                                           INPT2390
48 FORMAT (4(I3,F8.3,F9.4))                                               INPT2400
49 FORMAT (///,'LAMINATE PROPERTIES',//,1X,'LAMINATE TYPE',5X,'NO OF INPT2410
  1LAYERS    ',1X,'LAYER NUMBER',5X,'MATERIAL NO.',5X,'THICKNESS',5X,'INPT2420
  2 ORIENTATION'//)                                                      INPT2430
50 FORMAT (4X,I5,11X,I5)                                                  INPT2440
51 FORMAT (30X,I5,8X,I7,10X,F9.3,5X,F9.3)                                 INPT2450
52 FORMAT (5X,1I5,2G25.16,1G15.8)                                         INPT2460
53 FORMAT (5X,' MAX. NUMBER (',1I3,') OF ELEMENTS EXCEEDED, NEL=',1I4INPT2470
  1)                                                                      INPT2480
54 FORMAT (5X,' MAX. NUMB. (',1I3,') OF JOINTS EXCEEDED, NJT =',1I4) INPT2490
55 FORMAT (5X,' MAX. NUMB. (',1I2,') OF MATERIAL EXCEEDED, NMAT=',1I4INPT2500
  1)                                                                      INPT2510
56 FORMAT (5X,' MAX. NUMB. (',1I3,') OF C. LOADS EXC., NCLOAD =',1I4)INPT2520
57 FORMAT (5X,' MAX. N. (',1I3,') OF JOINTS WITH BOUND. COND. EXCDED,INPT2530
  1   NPBC =',1I4)                                                        INPT2540
58 FORMAT (5X,'MAX. NUMB.(',I5,') OF LAMINATE TYPES EXCEEDED,NLEL =',INPT2550
  1I5)                                                                    INPT2560
59 FORMAT (5X,'MAX. NUMB.(',I5,') OF LAMINATE LAYERS EXCEEDED,LYRNO =INPT2570
  1',I5)                                                                  INPT2580
   END                                                                    INPT2590
   SUBROUTINE MERGE (NPEL,NEQ)                                            MERG  10
   IMPLICIT REAL*8(A-H,O-Z)                                               MERG  20
   IMPLICIT INTEGER*2(I-N)                                                MERG  30
   COMMON /SOL/ BGK(434,86),ALOAD(434),ABGN                               MERG  40
   COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRMERG  50
  1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                              MERG  60
   COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLMERG  70
  1(182),THNP(217)                                                        MERG  80
   COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                        MERG  90
   COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                             MERG 100
   COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK                   MERG 110
   DIMENSION STK(24,24), AK(24,24), B(3,24)                               MERG 120
   REWIND 10                                                              MERG 130
   ZRO = 0.0D0                                                            MERG 140
C                                                                         MERG 150
   DO 1 I=1,36                                                            MERG 160
```

```
C                                                                 MERG 170
      DO 1 J=1,24                                                 MERG 180
      SS(I,J) = ZRO                                               MERG 190
    1 SN(I,J) = ZRO                                               MERG 200
C                                                                 MERG 210
C                                                                 MERG 220
      DC 6 IK=1,NEL                                               MERG 230
C                                                                 MERG 240
      DO 2 NN=1,NPEL                                              MERG 250
      LJT(NN) = NCON(IK,NN)                                       MERG 260
    2 LM(NN) = (NCON(IK,NN)-1)*2                                  MERG 270
C                                                                 MERG 280
      NS = 3*NPEL                                                 MERG 290
C                                                                 MERG 300
      DC 3 I1=1,NPEL                                              MERG 310
      I2 = LJT(I1)                                                MERG 320
C                                                                 MERG 330
      DO 3 J1=1,2                                                 MERG 340
    3 COORD(I1,J1) = COARD(I2,J1)                                 MERG 350
C                                                                 MERG 360
      INCIC = NCON(IK,15)                                         MERG 370
      IF (INDIC.NE.1) GO TO 4                                     MERG 380
      CALL LAMC (IK)                                              MERG 390
      GO TO 5                                                     MERG 400
    4 THETA = ANGL(IK)                                            MERG 410
      MATN = NCON(IK,14)                                          MERG 420
      INDIC = INDK(MATN)                                          MERG 430
      CALL STFMAT (ELAST,MATN,THETA,INDIC,NSTRES)                 MERG 440
      TK = ELCON(MATN,7)                                          MERG 450
    5 N = NCON(IK,13)*8                                           MERG 460
C    SET THICKNESS =1. FOR PLANE STRAIN                           MERG 470
      IF (NSTRES.EQ.1) TK=1.0D0                                   MERG 480
      CALL QUAD5 (STK,AK,B,NS,N,NGP)                              MERG 490
      WRITE (10) SS,SN                                            MERG 500
      IF (TK.EQ.0.0D0) TK=1.0D0                                   MERG 510
C                                                                 MERG 520
      CO 6 I=1,NPEL                                               MERG 530
C                                                                 MERG 540
      DC 6 J=1,NPEL                                               MERG 550
C                                                                 MERG 560
      DO 6 K=1,2                                                  MERG 570
      II = LM(I)+K                                                MERG 580
      KK = 2*(I-1)+K                                              MERG 590
C                                                                 MERG 600
      DC 6 L=1,2                                                  MERG 610
      JJ = LM(J)+L-II+1                                           MERG 620
      IF (JJ.LE.0) GO TO 6                                        MERG 630
      LL = 2*(J-1)+L                                              MERG 640
```

```
      BGK(II,JJ) = BGK(II,JJ)+STK(KK,LL)*TK
    6 CONTINUE
C
      SUM = ZRO
      CCN = ZRO
C
      DC 7 I=1,NEQ
    7 SLM = SUM+BGK(I,1)
C
      ABGN = SUM*1.0D20
      RETURN
      END
      SUBROUTINE LAMC (IK)
C     THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED
C     COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED
C     COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING
C     PROPERTIES ARE UNCOUPLED.IT CALCULATES AN AVERAGE STIFFNESS MATRIX
C     FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.
C     IK IS THE ELEMENT NUMBER
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCCN(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK
      DIMENSION C(3,3)
      LAMTP = NCON(IK,14)
      THETA = ANGL(IK)
      TK = 0.D0
C
      DC 1 I=1,3
C
      DC 1 J=1,3
    1 C(I,J) = 0.D0
C
      LYRS = LYRNO(LAMTP)
C
      DC 3 I=1,LYRS
      MATN = LAMAT(LAMTP,I)
      PHI = THETA+ORLAM(LAMTP,I)
      INDIC = INDK(MATN)
      CALL STFMAT (ELAST,MATN,PHI,INDIC,NSTRES)
      TK = TK+TKLAM(LAMTP,I)
C
      DC 2 J=1,3
```

```
C
      DC 2 K=1,3
    2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)
C
    3 CCNTINUE
C
C
      DO 4 J=1,3
C
      DO 4 K=1,3
    4 ELAST(J,K) = C(J,K)/TK
C
      RETURN
      END
      SUBROUTINE STFMAT (C,MATN,THETA,INDIC,NSTRES)
C  THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS MATRIX FOR A HOOKEAN
C  ELEMENT.THE SUBROUTINE CALCULATES AND STORES IN C(3X3)THE VALUES
C  OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER
C  EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS
C  EXHIBITING TWO DIMENSIONAL ORTHOTROPY.FOR ANISOTROPIC MATERIALS,
C  THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL
C  BASED L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM
C  BASED X-Y COORDINATE SYSTEM.
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      DIMENSION A(3,3), C(3,3)
C
      DC 1 I=1,3
C
      DO 1 J=1,3
      A(I,J) = 0.0D0
      C(I,J) = 0.0D0
    1 CCNTINUE
C
      IF (INCIC.EQ.1) GO TO 3
      IF (NSTRES.EQ.1) GO TO 2
C   FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS
C
      E = ELCCN(MATN,1)
      XNU = ELCON(MATN,2)
      EO = E/(1.0D0-XNU*XNU)
      C(1,1) = EO
      C(1,2) = EO*XNU
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,3) = EO*(1.0D0-XNU)/2.0D0
```

```
      RETURN
    2 CCNTINUE
C
C     FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN
C
      E = ELCCN(MATN,1)
      XNU = ELCON(MATN,2)
      E1 = E/((1.0D0+XNU)*(1.0D0-2.0D0*XNU))
      C(1,1) = E1*(1.0D0-XNU)
      C(1,2) = E1*XNU
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,3) = E1*(0.5D0-XNU)
      RETURN
    3 CCNTINUE
C
C     FCRM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASED
C     L-T CCORDINATE SYSTEM
C
      EL = ELCON(MATN,1)
      ET = ELCON(MATN,2)
      XNULT = ELCON(MATN,3)
      XNUTL = ELCON(MATN,4)
      GLT = ELCON(MATN,5)
      IF (EL.LT.-0.0D0) EL=XNULT*ET/XNUTL
      IF (ET.LT.-0.0D0) ET=EL*XNUTL/XNULT
      IF (XNULT.LT.-0.0D0) XNULT=EL*XNUTL/ET
      IF (XNUTL.LT.-0.0D0) XNUTL=XNULT*ET/EL
      Z = 1.0D0-XNULT*XNUTL
      A(1,1) = EL/Z
      A(1,2) = ET*XNULT/Z
      A(2,1) = A(1,2)
      A(2,2) = ET/Z
      A(3,3) = GLT
C
C      THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE
C      SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM.THE TRANSFORMATION IS
C      ACCCMPLISHED BY ROTATION ABOUT THE Z AXIS THRCUGH THETA DEGREES.
C
      PI = 3.14159265359D0
      THRAD = THETA*PI/180.0D0
      CCS1 = DCOS(THRAD)
      CCS2 = DCOS(THRAD)**2
      CCS3 = DCOS(THRAD)**3
      COS4 = DCOS(THRAD)**4
      SIN1 = DSIN(THRAD)
      SIN2 = DSIN(THRAD)**2
      SIN3 = CSIN(THRAD)**3
```

```
      SIN4 = DSIN(THRAD)**4
      TWO = 2.0D0
      FOUR = 4.0D0
      C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4
      C(1,2) = (A(1,1)+A(2,2)-FOUR*A(3,3))*SIN2*CCS2+A(1,2)*(SIN4+COS4)
      C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*(
     1A(3,3)))*SIN3*COS1
      C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4
      C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*A
     1(3,3))*SIN1*COS3
      C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3)))*SIN2*COS2+A(3,3)*(SIN
     14+COS4)
      C(2,1) = C(1,2)
      C(3,1) = C(1,3)
      C(3,2) = C(2,3)
      RETURN
      END
      SUBROUTINE QUAD5 (STK,AK,B,NS,N,NGP)
C
C    QUAD5 IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE STIFFNESS
C    MATRIX FOR ANY OF THE THREE QUADRILATERAL ISOPARAMETRIC
C    ELEMENTS USED IN THIS PROGRAM.
C    GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.
C
C         STK  IS AN ARRAY DIMENSIONED NXN THAT WILL CONTAIN THE STIFFNES
C              MATRIX UPON RETURN TO THE CALLING PROGRAM
C         SS AND SN ARE THE STRESS AND STRAIN VERSUS NODAL POINT
C              DISPLACEMENT TRANSFORMATION MATRICES, BOTH ARE DIMENSIONED
C              NSXN WHERE NS= 3*NPT AND N= 2*NPT
C         AK   IS AN ARRAY DIMENSIONED NXN REQUIRED IN FORMK
C         B    IS AN ARRAY DIMENSIONED 2XN REQUIRED IN FORMK
C
C         NGP IS THE NUMBER OF GAUSS POINTS USED IN THE INTEGRATION
C
C         COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL
C                     THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED
C         ELAST(3,3)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS
C                     OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS
C                     IS: SIGMAX,SIGMAY,TAUXY
C
C
C              ETA                    ETA                   ETA
C               +                      +                     +
C          U2----+-----U1         U3----U2----U1        U4--U3+-U2--U1
C          |    +      |          |      +     |        |     +     |
C          |    +      |          |      +     |        |     +     |
C          |    +      |          |      +     |        |     +     |
C          |    +      |          |      +     |        U5    +     U12
```

```
C              +         |        |         +      |        |              +        |
C              ++++++++++XI       U4        ++++++U8++XI    |              ++++++++++XI
C                        |        |                 |       U6                      |
C                        |        |                 |       |                       U11
C                        |        |                 |       |                       |
C                        |        |                 |       |                       |
C                        |        |                 |       |                       |
C        U3-----------U4          U5----U6----U7            U7--U8--U9--U10
C
C     CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL
C
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT  INTEGER*2(I-N)
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK
      DIMENSION AK(24,24), STK(24,24), B(3,24)
      DIMENSION XI(5), AI(5), AIA(5,5)
      DIMENSION X2(2), X3(3), X4(4), X5(5), A2(2), A3(3), A4(4), A5(5)
      DIMENSION XYL(4,2), XYQ(8,2), XYC(12,2)
      DATA X2/.577350269189626D0,-.577350269189626D0/
      DATA A2/1.0D0,1.0D0/
      DATA X3/.774596669241483D0,0.0D0,-.774596669241483D0/
      DATA A3/.555555555555556D0,.888888888888889D0,.555555555555556D0/
      DATA X4/.861136311594D0,.339981043584 8D0,-.339981043584 8D0,-.8611
     1363115940D0/
      DATA A4/.347854845137 4D0,.652145154862 5D0,.652145154862 5D0,.347854
     18451374D0/
      DATA X5/.906179845938 6D0,.538469310105 6D0,0.0D0,-.538469310105 6D0,
     1-.9061798459386D0/
      DATA A5/.236926885056 1D0,.478628670499 3D0,.568888888888 8D0,.478628
     16704993D0,.2369268850561D0/
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,
     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/
      DATA XYC/1.0D0,.333333333333333D0,-.333333333333333D0,-1.0D0,-1.0D0,-1
     1.0D0,-1.0D0,-.333333333333333D0,.333333333333333D0,1.0D0,1.0D0,1.0D0,1
     2.0D0,1.0D0,1.0D0,1.0D0,.333333333333333D0,-.333333333333333D0,-1.0D0,-
     31.0D0,-1.0D0,-1.0D0,-.333333333333333D0,.333333333333333D0/
      IF (NGP.EQ.0) NGP = 3
C
      DO 1 I=1,N
C
      DO 1 J=1,N
    1 STK(I,J) = 0.0D0
C
      JUMP = NGP-1
C
      DO 6 I=1,NGP
      GO TO (2,3,4,5), JUMP
```

```
    2 XI(I)  =  X2(I)                                                   QUAD 800
      AI(I)  =  A2(I)                                                   QUAD 810
      GO TO 6                                                           QUAD 820
    3 XI(I)  =  X3(I)                                                   QUAD 830
      AI(I)  =  A3(I)                                                   QUAD 840
      GC TO 6                                                           QUAD 850
    4 XI(I)  =  X4(I)                                                   QUAD 860
      AI(I)  =  A4(I)                                                   QUAD 870
      GO TO 6                                                           QUAD 880
    5 XI(I)  =  X5(I)                                                   QUAD 890
      AI(I)  =  A5(I)                                                   QUAD 900
    6 CCNTINUE                                                          QUAD 910
C                                                                       QUAD 920
C                                                                       QUAD 930
      DO 7 I=1,NGP                                                      QUAD 940
C                                                                       QUAD 950
      DC 7 J=1,NGP                                                      QUAD 960
    7 AIA(I,J) = AI(I)*AI(J)                                            QUAD 970
C                                                                       QUAD 980
C                                                                       QUAD 990
      DO 8 I=1,NGP                                                      QUAD1000
      X = XI(I)                                                         QUAD1010
C                                                                       QUAD1020
      DC 8 J=1,NGP                                                      QUAD1030
      Y = XI(J)                                                         QUAD1040
      CALL FORMK (AK,X,Y,B,N)                                           QUAD1050
      AIAIJ = AIA(I,J)                                                  QUAD1060
C                                                                       QUAD1070
      DC 8 K=1,N                                                        QUAD1080
C                                                                       QUAD1090
      DO 8 L=1,N                                                        QUAD1100
    8 STK(K,L) = STK(K,L)+AIAIJ*AK(K,L)                                 QUAD1110
C                                                                       QUAD1120
C                                                                       QUAD1130
      DC 9 I=1,N                                                        QUAD1140
C                                                                       QUAD1150
      DO 9 J=I,N                                                        QUAD1160
      STK(I,J) = (STK(I,J)+STK(J,I))*0.5D0                              QUAD1170
    9 STK(J,I) = STK(I,J)                                               QUAD1180
C                                                                       QUAD1190
      NPT = N/2                                                         QUAD1200
      IGO = NPT/4                                                       QUAD1210
C                                                                       QUAD1220
      DO 16 I=1,NPT                                                     QUAD1230
      GO TO (10,11,12), IGO                                             QUAD1240
   10 X = XYL(I,1)                                                      QUAD1250
      Y = XYL(I,2)                                                      QUAD1260
      GC TO 13                                                          QUAD1270
```

```
   11 X = XYQ(I,1)                                                      QUAD1280
      Y = XYQ(I,2)                                                      QUAD1290
      GC TO 13                                                          QUAD1300
   12 X = XYC(I,1)                                                      QUAD1310
      Y = XYC(I,2)                                                      QUAD1320
   13 CALL FORMK (AK,X,Y,B,N)                                           QUAD1330
C                                                                       QUAD1340
      CC 14 L=1,3                                                       QUAD1350
C                                                                       QUAD1360
      DO 14 M=1,N                                                       QUAD1370
      AK(L,M) = 0.0D0                                                   QUAD1380
C                                                                       QUAD1390
      DO 14 NN=1,3                                                      QUAD1400
   14 AK(L,M) = AK(L,M)+ELAST(L,NN)*B(NN,M)                             QUAD1410
C                                                                       QUAD1420
      II = 3*(I-1)                                                      QUAD1430
C                                                                       QUAD1440
      DO 15 J=1,3                                                       QUAD1450
      IJ = II+J                                                         QUAD1460
C                                                                       QUAD1470
      DO 15 K=1,N                                                       QUAD1480
      SS(IJ,K) = AK(J,K)                                                QUAD1490
   15 SN(IJ,K) = B(J,K)                                                 QUAD1500
C                                                                       QUAD1510
   16 CCNTINUE                                                          QUAD1520
C                                                                       QUAD1530
      RETURN                                                            QUAD1540
      END                                                               QUAD1550
      SLBROUTINE FORMK (AK,X,Y,B,N)                                     FRMK  10
C  FCRMK FORMS THE STIFFNESS MATRIX AND THE  B MATRIX AS FUNCTIONS      FRMK  20
C  OF XI AND ETA FOR THE ISOPARAMETRIC ELEMENTS USED IN THIS PROGRAM    FRMK  30
C                                                                       FRMK  40
      IMPLICIT REAL*8(A-H,O-Z)                                          FRMK  50
      IMPLICIT INTEGER*2(I-N)                                           FRMK  60
      DIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24), B1(24,FRMK  70
     13), AK(24,24)                                                     FRMK  80
      CCMMCN COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK              FRMK  90
      ZERO = 0.0D0                                                      FRMK 100
C                                                                       FRMK 110
      DO 1 I=1,3                                                        FRMK 120
C                                                                       FRMK 130
      DO 1 J=1,N                                                        FRMK 140
    1 B(I,J) = ZERO                                                     FRMK 150
C                                                                       FRMK 160
      CNE = 1.0D0                                                       FRMK 170
      TWO = 2.0D0                                                       FRMK 180
      FOUR = 4.0D0                                                      FRMK 190
      NFT = N/2                                                         FRMK 200
```

```
C
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA
C
      IGC = N/8
      GO TO (2,3,4), IGO
C
C     LINEAR FUNCTIONS
C
    2 W1(1,3) = -(ONE-Y)/FOUR
      W1(1,4) = -W1(1,3)
      W1(1,1) = (ONE+Y)/FOUR
      W1(1,2) = -W1(1,1)
      W1(2,3) = -(ONE-X)/FOUR
      W1(2,4) = -(ONE+X)/FOUR
      W1(2,1) = -W1(2,4)
      W1(2,2) = -W1(2,3)
      GO TO 5
    3 CONTINUE
C
C     QUADRATIC FUNCTIONS
C
      TXPY = TWO*X+Y
      TXMY = TWO*X-Y
      TYPX = TWO*Y+X
      TYMX = TWO*Y-X
      CMY = ONE-Y
      OPY = ONE+Y
      OPX = ONE+X
      OMX = ONE-X
      W1(1,5) = OMY*TXPY/FOUR
      W1(1,6) = -OMY*X
      W1(1,7) = OMY*TXMY/FOUR
      W1(1,8) = OPY*OMY/TWO
      W1(1,1) = OPY*TXPY/FOUR
      W1(1,2) = -OPY*X
      W1(1,3) = OPY*TXMY/FOUR
      W1(1,4) = -OPY*OMY/TWO
      W1(2,5) = OMX*TYPX/FOUR
      W1(2,6) = -OPX*OMX/TWO
      W1(2,7) = OPX*TYMX/FOUR
      W1(2,8) = -Y*OPX
      W1(2,1) = OPX*TYPX/FOUR
      W1(2,2) = OPX*OMX/TWO
      W1(2,3) = OMX*TYMX/FOUR
      W1(2,4) = -Y*OMX
      GO TO 5
    4 CONTINUE
C
```

```
C     CUBIC FUNCTIONS
C
       OPX = ONE+X
       OPY = ONE+Y
       OMX = ONE-X
       OMY = ONE-Y
       TPTMNX = 3.0D0+2.0D0*X-9.0D0*X*X
       TMTMNX = 3.0D0-2.0D0*X-9.0D0*X*X
       TPTMNY = 3.0D0+2.0D0*Y-9.0D0*Y*Y
       TMTMNY = 3.0D0-2.0D0*Y-9.0D0*Y*Y
       TYPO = 3.0D0*Y+ONE
       TYMO = 3.0D0*Y-ONE
       TXPO = 3.0D0*X+ONE
       TXMO = 3.0D0*X-ONE
       TMTPEX = 10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y
       TMTMEX = 10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y
       TMNPEY = 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y
       TMNMEY = 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y
       TTT = 32.0D0
       TTN = 32.0D0/9.0D0
       W1(1,7) = OMY*TMTPEX/TTT
       W1(1,8) = -TPTMNX*OMY/TTN
       W1(1,9) = TMTMNX*OMY/TTN
       W1(1,10) = -OMY*TMTMEX/TTT
       W1(1,11) = -OPY*OMY*TYMO/TTN
       W1(1,12) = OPY*OMY*TYPO/TTN
       W1(1,1) = -OPY*TMTMEX/TTT
       W1(1,2) = TMTMNX*OPY/TTN
       W1(1,3) = -TPTMNX*OPY/TTN
       W1(1,4) = OPY*TMTPEX/TTT
       W1(1,5) = -OPY*OMY*TYPO/TTN
       W1(1,6) = OPY*OMY*TYMO/TTN
       W1(2,7) = OMX*TMNPEY/TTT
       W1(2,8) = OPX*OMX*TXMO/TTN
       W1(2,9) = -OPX*OMX*TXPO/TTN
       W1(2,10) = OPX*TMNPEY/TTT
       W1(2,11) = -TPTMNY*OPX/TTN
       W1(2,12) = TMTMNY*OPX/TTN
       W1(2,1) = -OPX*TMNMEY/TTT
       W1(2,2) = OPX*OMX*TXPO/TTN
       W1(2,3) = -OPX*OMX*TXMO/TTN
       W1(2,4) = -OMX*TMNMEY/TTT
       W1(2,5) = OMX*TMTMNY/TTN
       W1(2,6) = -OMX*TPTMNY/TTN
     5 CONTINUE
C
C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)
C
```

```
C
      DC 6 I=1,2
C
      DO 6 J=1,2
      AJ(I,J) = ZERO
C
      DO 6 K=1,NPT
    6 AJ(I,J) = AJ(I,J)+W1(I,K)*COORD(K,J)
C
C
C CALCULATE DETERMINANT OF THE JACOBIAN
C
      DTJ = AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
C
C INVERT JACOBIAN IN AJIN
C
      AJIN(1,1) = AJ(2,2)/DTJ
      AJIN(1,2) = -AJ(1,2)/DTJ
      AJIN(2,1) = -AJ(2,1)/DTJ
      AJIN(2,2) = AJ(1,1)/DTJ
C
C FCRM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y
C
C
      DC 7 I=1,2
C
      DO 7 J=1,NPT
      DNX(I,J) = ZERO
C
      DC 7 K=1,2
    7 DNX(I,J) = DNX(I,J)+AJIN(I,K)*W1(K,J)
C
C
C FORM B(I,J) MATRIX
C
C
      DC 8 I=1,NPT
      IOD = 2*I-1
      IEV = 2*I
      B(1,IOD) = DNX(1,I)
      B(2,IEV) = DNX(2,I)
      B(3,IEV) = DNX(1,I)
    8 B(3,IOD) = DNX(2,I)
C
C
C FORM STIFFNESS BY THE CONGRUENT TRANSFORMATION  BT*D*B
C
C
```

```
      DO 9 I=1,N
C
      DO 9 J=1,3
      B1(I,J) = ZERO
C
      DO 9 K=1,3
    9 B1(I,J) = B1(I,J)+B(K,I)*ELAST(K,J)
C
C
      DO 10 I=1,N
C
      DO 10 J=1,N
      AK(I,J) = ZERO
C
      DO 10 K=1,3
   10 AK(I,J) = AK(I,J)+B1(I,K)*B(K,J)
C
C
      DO 11 I=1,N
C
      DO 11 J=I,N
      AK(I,J) = ((AK(I,J)+AK(J,I))/2.0D0)*DTJ
   11 AK(J,I) = AK(I,J)
C
      RETURN
      END
      SUBROUTINE LDLT (N,M)
C ***********************************************************************
C    THIS SUBROUTINE DECOMPOSES THE COEFFICIENT MATRIX A* OF THE LINEAR
C    BANDED SYMMETRICAL SYSTEM (A*)(X) = (B), INTO THE PRODUCT (L)(D)(LT)
C    THE UPPER BAND ONLY IS USED, AND IT MUST BE STORED IN A RECTANGULAR
C    ARRAY (A) WITH ALL ITS DIAGONAL ELEMENTS IN THE FIRST COLUMN
C    MATRIX (A) IS DESTROYED IN THE PROCESS AND THE RESULTS RETURNED IN
C    THE SAME ARRAY. MATRIX (D) IS PUT IN THE FIRST COLUMN AND THE REST
C    OF THE MATRIX CONTAINS THE ELEMENTS OF THE UNIT TRIANGULAR MATRIX
C    IN BANDED FORM WITHOUT ITS IMPLIED UNIT DIAGONAL ELEMENTS.
C
C         A  IS THE NAME OF THE COEFFICIENT MATRIX
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM
C         M  IS THE HALF BAND WIDTH OF THE SYSTEM
C
C    CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972
C ***********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /SOL/ A(434,86),B(434),ABGN
      AN = N
      APZRO = ABGN*1.0D-30/AN
```

```
      AVSN = APZRO/ABGN
      ZRC = 0.0D0
      NM = N-1
C
      DC 3 I=1,NM
      DIAG = A(I,1)
      IF (DIAG.EQ.ZRO) GO TO 4
      IF (DIAG.LE.APZRO) WRITE (6,6) I
C
      DO 1 J=2,M
      IF (DABS(A(I,J)).LE.AVSN) A(I,J)=0.0D0
    1 A(I,J) = A(I,J)/DIAG
C
C
      DC 3 J=2,M
      L = I+J-1
      IF (L.GT.N) GO TO 3
      AA = A(I,J)*DIAG
      IF (AA.EQ.ZRO) GO TO 3
C
      DO 2 K=J,M
      ML = 1+K-J
    2 A(L,ML) = A(L,ML)-AA*A(I,K)
C
    3 CCNTINUE
C
      RETURN
    4 WRITE (6,5) I
      STOP
C
    5 FORMAT (//,5X,' MATRIX IS SINGULAR ',1I5)
    6 FCRMAT (//,5X,' WARNING  MATRIX IS NEARLY SINGULAR  EXFCUTION CONT
     1INUES ', 1I5,/,5X,' RESIDUAL ITERATION MAY YIELD ERRONEOUS RESULTS
     2   '//)
      END
      SUBROUTINE SOLV (N,M)
C ************************************************************************
C   THIS SUBROUTINE PERFORMS A FORWARD SUBSTITUTION, FOLLOWED BY A
C   BACKWARD SUBSTITUTION FOR THE SYSTEM (A*) X= (B), AFTER THE
C   MATRIX (A) HAS BEEN DECOMPOSED BY THE SUBROUTINE LDLT
C
C        A  IS THE NAME OF THE DECOMPOSED MATRIX CF COEFFICIENT
C        B  IS THE NAME OF THE COLUMN LOAD VECTOR OR RIGHT HAND MEMBER
C           THE ANSWERS ARE RETURNED IN THIS VECTCR UPON EXIT
C        N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM
C        M  IS THE HALF BAND WIDTH OF THE SYTEM
C
C   AFTER ONE CALL TO LDLT,ONE CALL OF SOLV PER LOAD VECTOR IS REQ'D.
```

```
C    CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972
C ***********************************************************************
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /SOL/ A(434,86),B(434),ABGN
      NM = N-1
C
      DO 1 I=1,NM
      BI = B(I)
C
      DO 1 J=2,M
      L = I+J-1
      IF (L.GT.N) GO TO 1
      B(L) = B(L)-A(I,J)*BI
    1 CONTINUE
C
C
      DO 2 I=1,N
    2 B(I) = B(I)/A(I,1)
C
C
      DO 3 L=2,N
      IR = N-L+1
      BIRP = B(IR+1)
C
      DO 3 J=2,M
      IRO = IR-J+2
      IF (IRO.LE.0) GO TO 3
      B(IRO) = B(IRO)-A(IRO,J)*BIRP
    3 CONTINUE
C
      RETURN
      END
      SUBROUTINE STRESS (NPEL)
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /SOL/ BGK(434,86),DISP(434),ABGN
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK
      DIMENSION SSJNT(217,7), SNJNT(217,7), SSEL(36), SNEL(36), DSPEL(36
     1)
      DIMENSION TEEL(12)
      EQUIVALENCE (BGK(1,1),SSJNT(1,1)), (BGK(1,5),SNJNT(1,1)), (BGK(1,9
     1),SSEL(1)), (BGK(1,10),SNEL(1)), (BGK(1,11),DSPEL(1))
      PI = 3.14159265359D0
```

```
      ZRC = 0.0D0
      REWIND 10
C
      DC 1 I=1,NJT
C
      DO 1 J=1,7
      SNJNT(I,J) = ZRO
    1 SSJNT(I,J) = ZRO
C
C
      DO 11 I=1,NEL
      READ (10) SS,SN
      IF (NTLD.EQ.0) GO TO 3
      MATN = NCON(I,14)
      YMOD = ELCON(MATN,1)
      PRAT = ELCON(MATN,2)
      ALPH = ELCON(MATN,3)
      IF (NSTRES.EQ.0) GO TO 2
      CCNST = ALPH*YMOD/(1.0D0-2.0D0*PRAT)
      GO TO 3
    2 CONST = ALPH*YMOD/(1.0D0-PRAT)
    3 CONTINUE
C
      DO 4 J=1,NPEL
      NCIJ = NCON(I,J)
      JA = NCIJ*2-1
      JJ = J*2-1
      IF (NTLD.NE.0) TEEL(J)=THNP(NCIJ)
      DSPEL(JJ) = DISP(JA)
      JJ = JJ+1
      JA = JA+1
    4 DSPEL(JJ) = DISP(JA)
C
      NJLM = NPEL*2
      NPELT = NPEL*3
C
      DC 5 I1=1,NPELT
      SSEL(I1) = ZRO
      SNEL(I1) = ZRO
C
      DO 5 J1=1,NJLM
      SSEL(I1) = SSEL(I1)+SS(I1,J1)*DSPEL(J1)
    5 SNEL(I1) = SNEL(I1)+SN(I1,J1)*DSPEL(J1)
C
      IF (NTLC.EQ.0) GO TO 7
C
      DO 6 J=1,NPEL
      JJ0 = J*3-2
```

```
      JJD = JJO+1
      CCRT = CONST*TEEL(J)
      SSEL(JJO) = SSEL(JJO)-CORT
      SSEL(JJD) = SSEL(JJD)-CORT
    6 CCNTINUE
C
    7 CONTINUE
      IF (NMAT.EQ.1) GO TO 9
      WRITE (6,19) I
      ILO = 0
      IUP = 0
C
      DC 8 J=1,NPEL
      IJT = NCON(I,J)
      ILC = IUP+1
      IUP = ILO+2
    8 WRITE (6,18) IJT,(SSEL(K),K=ILO,IUP),(SNEL(L),L=ILO,IUP)
C
    9 CCNTINUE
      ICT = 0
C
      DO 11 I2=1,NPEL
      I2A = NCON(I,I2)
C
      DO 10 J2=1,3
      ICT = ICT+1
      SSJNT(I2A,J2) = SSEL(ICT)+SSJNT(I2A,J2)
   10 SNJNT(I2A,J2) = SNEL(ICT)+SNJNT(I2A,J2)
C
   11 SNJNT(I2A,7) = SNJNT(I2A,7)+1.
C
C
      DC 15 I3=1,NJT
      SCNT = SNJNT(I3,7)
C
      DC 12 J3=1,3
      SSJNT(I3,J3) = SSJNT(I3,J3)/SCNT
   12 SNJNT(I3,J3) = SNJNT(I3,J3)/SCNT
C
      GAU = SSJNT(I3,3)
      SXPSY = (SSJNT(I3,1)+SSJNT(I3,2))/2.0D0
      SXMSY = (SSJNT(I3,1)-SSJNT(I3,2))/2.0D0
      AINT = DSQRT(SXMSY*SXMSY+GAU*GAU)
      SSJNT(I3,4) = SXPSY+AINT
      SSJNT(I3,5) = SXPSY-AINT
      SSJNT(I3,6) = (SSJNT(I3,4)-SSJNT(I3,5))/2.0D0
      IF (DABS(SXMSY).LE.1.D-10) GO TO 13
      RAN = DATAN2(GAU,SXMSY)
```

```
      GC TO 14                                                          STRS1120
   13 RAN = DSIGN(PI,GAU)/2.0D0                                         STRS1130
   14 SSJNT(I3,7) = RAN*180.0D0/(2.0D0*PI)                              STRS1140
      GAU = SNJNT(I3,3)                                                 STRS1150
      SXPSY = (SNJNT(I3,1)+SNJNT(I3,2))/2.0D0                           STRS1160
      SXMSY = (SNJNT(I3,1)-SNJNT(I3,2))/2.0D0                           STRS1170
      AINT = DSQRT(SXMSY*SXMSY+GAU*GAU)                                 STRS1180
      SNJNT(I3,4) = SXPSY+AINT                                          STRS1190
      SNJNT(I3,5) = SXPSY-AINT                                          STRS1200
      SNJNT(I3,6) = (SNJNT(I3,4)-SNJNT(I3,5))/2.0D0                     STRS1210
   15 SNJNT(I3,7) = SSJNT(I3,7)                                         STRS1220
C                                                                       STRS1230
      WRITE (6,21)                                                      STRS1240
C                                                                       STRS1250
      DC 16 I=1,NJT                                                     STRS1260
   16 WRITE (6,20) I,COARD(I,1),COARD(I,2),(SSJNT(I,J),J=1,7)           STRS1270
C                                                                       STRS1280
      WRITE (6,22)                                                      STRS1290
C                                                                       STRS1300
      DC 17 I=1,NJT                                                     STRS1310
   17 WRITE (6,20) I,COARD(I,1),COARD(I,2),(SNJNT(I,J),J=1,6),SSJNT(I,7)STRS1320
C                                                                       STRS1330
      RETURN                                                            STRS1340
C                                                                       STRS1350
   18 FORMAT (5X,I5,6G12.4)                                             STRS1360
   19 FORMAT (//,2X,' S-T-R-E-S-S-E-S  /  S-T-R-A-I-N-S  FOR  ELEMENT',ISTRS1370
     13,       //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'EPS STRS1380
     2X',7X,'EPS Y', 7X,'GAMMA ',//)                                    STRS1390
   20 FCRMAT (5X,I5,8G12.4,5X,1G12.4)                                   STRS1400
   21 FORMAT (///,1X,' A-V-E-R-A-G-E  S-T-R-E-S-S-E-S  AT  THE  JOINTS',STRS1410
     1 //,5X,' JOINT',' X COORD',5X,'Y COORD',5X,'SIG  X ',6X,'SIG Y ',STRS1420
     27X,'TAU ',7X,'S.MAX',7X,'S.MIN',7X,'T MAX ',12X,'  PHI(DEG.)',//) STRS1430
   22 FCRMAT (///,1X,' A-V-E-R-A-G-E  S-T-R-A-I-N-S  AT  THE  JOINTS',  STRS1440
     1 //,5X,' JOINT',' X COORD',5X,'Y COORD',5X,'EPS  X ',6X,'EPS Y ',STRS1450
     27X,'GAMMA',6X,'E.MAX',7X,'E.MIN',7X,'G.MAX ',12X,'  PHI(DEG.)',//)STRS1460
      END                                                               STRS1470
      SUBROUTINE BCOND (NBAND)                                          BCND  10
      IMPLICIT REAL*8(A-H,O-Z)                                          BCND  20
      IMPLICIT INTEGER*2(I-N)                                           BCND  30
      CCMMON /SOL/ BGK(434,86),ALOAD(434),ABGN                          BCND  40
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRBCND  50
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL                         BCND  60
      CCMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLBCND  70
     1(182),THNP(217)                                                   BCND  80
      PI = 3.14159265359D0                                              BCND  90
      NEQ = NJT*2                                                       BCND 100
C                                                                       BCND 110
      DC 16 I=1,NPBC                                                    BCND 120
```

```
      IJT = NBC(I,1)
      IEQ = IJT*2-1
      KCDE = NBC(I,2)
      ICIA = 0
      GC TO (1,3,4,6,10,11,13), KODE
    1 IEQ = IEQ+1
    2 BGK(IEQ,1) = ABGN
      IF (IDIA.EQ.1) GO TO 5
      GC TO 16
    3 GO TO 2
    4 IDIA = 1
      GC TO 2
    5 IDIA = 0
      GO TO 1
    6 DXY = CLOAD(IJT,2)
      IEG = IEQ+1
    7 ALOAD(IEQ) = ALOAD(IEQ)-BGK(IEQ,1)*DXY
      BGK(IEQ,1) = ABGN
C
      DC 9 J=2,NBAND
      IROW = IEQ-J+1
      ICOL = IEC-1+J
      IF (IROW.LT.1) GO TO 8
      ALOAD(IROW) = ALOAD(IROW)-BGK(IROW,J)*DXY
    8 IF (ICOL.GT.NEQ) GO TO 9
      ALCAD(ICOL) = ALOAD(ICOL)-BGK(IEQ,J)*DXY
    9 CCNTINUE
C
      IF (IDIA.EQ.1) GO TO 12
      GC TO 16
   1C DXY = CLOAD(IJT,1)
      GO TO 7
   11 IDIA = 1
      DXY = CLOAD(IJT,1)
      GC TO 7
   12 IDIA = 0
      GC TO 6
   13 ALPHA = CLOAD(IJT,2)*PI/180.0D0
      CAL = DCOS(ALPHA)
      SAL = DSIN(ALPHA)
      IECP = IEQ+1
C
      DC 15 J=3,NBAND
      IROW = IEQ-J+2
      JM = J-1
      IF (IROW.LT.1) GO TO 14
      A1 = BGK(IROW,JM)*CAL+BGK(IROW,J)*SAL
      A2 = BGK(IROW,J)*CAL-BGK(IROW,JM)*SAL
```

```
      BGK(IROW,JM) = A1
      BGK(IROW,J) = A2
   14 IF ((IEQ+J-1).GT.NEQ) GO TO 15
      A3 = BGK(IEQ,J)*CAL+BGK(IEQP,JM)*SAL
      A4 = BGK(IEQP,JM)*CAL-BGK(IEQ,J)*SAL
      BGK(IEQ,J) = A3
      BGK(IEQP,JM) = A4
   15 CCNTINUE
C
      A1 = BGK(IEQ,1)*CAL*CAL+BGK(IEQ,2)*CAL*SAL*2.0+BGK(IEQP,1)*SAL*SAL
      A2 = BGK(IEQ,2)*(CAL*CAL-SAL*SAL)+(BGK(IEQP,1)-BGK(IEQ,1))*SAL*CAL
      BGK(IEQ,1) = A1
      BGK(IEQ,2) = A2
      BGK(IEQP,1) = ABGN
   16 CCNTINUE
C
      RETURN
      END
      SUBROUTINE DISPL
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CCMMON /SOL/ BGK(434,86),DISP(434),ABGN
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
     1(182),THNP(217)
      CCMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
      DIMENSICN REACT(218,2)
      ECUIVALENCE (BGK(1,1),REACT(1,1))
      PI = 3.14159265359D0
C
      DO 11 I=1,NPBC
      IJT = NBC(I,1)
      IEQ = IJT*2-1
      KODE = NBC(I,2)
      ICIA = 0
      DPBC = 0.0D0
      GC TO (1,3,4,6,7,8,10), KODE
    1 IEQ = IEQ+1
      JR = 2
    2 REACT(I,JR) = -DISP(IEQ)*ABGN
      DISP(IEQ) = DPBC
      IF (IDIA.EQ.1) GO TO 5
      IF (IDIA.EQ.2) GO TO 9
      GO TO 11
    3 JR = 1
      GC TO 2
    4 IDIA = 1
```

```
      JR = 1
      GO TO 2
    5 ICIA = 0
      GC TO 1
    6 DPBC = CLOAD(IJT,2)
      GO TO 1
    7 JR = 1
      DPBC = CLOAD(IJT,1)
      GC TO 2
    8 ICIA = 2
      JR = 1
      DPBC = CLOAD(IJT,1)
      GC TO 2
    9 ICIA = 0
      DPBC = CLOAD(IJT,2)
      GO TO 1
   10 ALPHA = CLOAD(IJT,2)*PI/180.0D0
      CAL = DCOS(ALPHA)
      SAL = DSIN(ALPHA)
      A1 = DISP(IEQ)*CAL
      A2 = DISP(IEQ)*SAL
      IEQP = IEQ+1
      A3 = -DISP(IEQP)*SAL*ABGN
      A4 = DISP(IEQP)*CAL*ABGN
      DISP(IEQ) = A1
      DISP(IEQP) = A2
      REACT(I,1) = -A3
      REACT(I,2) = -A4
   11 CONTINUE
C
      MJP = MJT+1
      REACT(MJP,1) = 0.0D0
      REACT(MJP,2) = 0.0D0
C
      DC 12 I=1,NPBC
      REACT(MJP,1) = REACT(MJP,1)+REACT(I,1)
   12 REACT(MJP,2) = REACT(MJP,2)+REACT(I,2)
C
      RETURN
      END
```

APPENDIX B

```
C   PROGRAM ISLOAD
C    ***********************************************************************
C    THIS SERVICE PROGRAM FORMS CONSISTENT LOAD VECTORS FOR ANY OF THE
C    FINITE ELEMENTS OF THE PROGRAM  ISANIS
C    ***********************************************************************
C
C    THREE DIFFERENT LOAD VECTORS CAN BE FORMED.
C    (A) THERMAL LOAD CORRESPONDING TO A TEMPERATURE DISTRIBUTION
C    (B) BODY FORCE LOAD/UNIT VOLUME DISTRIBUTED THROUGH AN ELEMENT
C    (C) INITIAL STRETCH OR CONTRACTION OF AN ELEMENT
C
C    THE LOAD INTENSITY FUNCTION OVER AN ELEMENT IS DETERMINED BY
C    ITS NODAL VALUES AND THE SAME SHAPE FUNCTIONS USED IN THE
C    CONSTRUCTION OF THE ELEMENT ITSELF.  THUS A 4 NODED ELEMENT
C    WILL ALLOW LINEAR VARIATION, AN 8 NODED ELEMENT QUADRATIC VARIATION
C    AND A 12 NODED ELEMENT CUBIC VARIATION
C
C    THIS PROGRAM IS AN EXPANSION OF THE PLISOP LOAD GENERATOR
C    CODED BY GILLES CANTIN IN 1972
C    JAMES M. GILL, MARCH 1975
C   INPUT CARCS NEEDED
C
C   TITLE                      (20A4)          ONE CARD PER PROBLEM
C
C   PROBLEM PARAMETERS        (12I5)          ONE CARD PER PROBLEM
C
C    COL.    VARIABLE
C     1-5        NEL   NUMBER OF ELEMENTS (MAX. IS 182)
C     6-10       NJT   NUMBER OF JOINTS (MAX. IS 217)
C    11-15       NMAT  NUMBER OF MATERIALS (MAX. IS 10)
C    16-20       NCLOAD  NUMBER OF CONCENTRATED LOADS
C    21-25       NPBC   NUMBER OF JOINTS WITH BOUNDARY CONDITIONS
C    26-30       NSTRES   ISOTROPIC MATERIAL PROBLEM TYPE.NSTRES=0 MEANS
C                         PLANE STRESS, NSTRES = 1 MEANS PLANE STRAIN
C    31-35       NGP   NUMBER OF GAUSS POINTS DESIRED IN THE INTEGRATION
C    36-40       NTLD  1  IF A THERMAL LOAD IS PRESENT  0  OTHERWISE
C    41-45       NTBY  1  IF A BODY FORCE LOAD IS PRESENT  0 OTHERWISE
C    46-50       NTIN  1  IF AN INITIAL STRETCH IS TO BE CONSIDERED
C    51-55       NCARD  MUST BE  1  IF A PUNCED LOAD  VECTOR SUITABLE FOR
C                USE WITH ISANIS IS DESIRED
C    56-60       NLEL  NUMBER OF LAMINATE TYPES WHOSE MATERIAL
C                      PROPERTIES ARE TO BE CALCULATED FROM
C                      INPUT LAMINA PROPERTIES.  (MAX. 50)
C    JOINT CARDS      (I5,5X,4G15.8,I5)             NJT CARDS PER PROBLEM
```

```
C     COLUMN                  VARIABLE                                   MAIN 450
C        1-5                  JOINT NUMBER                               MAIN 460
C      11-25                  X COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 470
C                             RADIUS VECTOR LENGTH (POLAR COORD.)        MAIN 480
C      26-40                  Y-COORDINATE OF THE JOINT (CARTESIAN COORD)MAIN 490
C                             POLAR ANGLE IN DEGREES (POLAR COORD.)      MAIN 500
C      41-55                  X-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 510
C      56-70                  Y-COMPONENT OF THE LOAD AT THE JOINT (CART)MAIN 520
C      70-75                  COORDINATE SYSTEM INDICATOR. IND. = ZERO IFMAIN 530
C                             THE COORDINATE SYSTEM IS CARTESIAN, IF IND MAIN 540
C                             IS NOT ZERO, THE COORDINATE SYSTEM IS POLARMAIN 550
C                                                                        MAIN 560
C     MATERIAL PROPERTIES CARDS(2I5,7F10.2)      NMAT CARDS PER PROBLEM  MAIN 570
C      COLUMN             VARIABLE                                       MAIN 580
C        1-5              MATERIAL NUMBER                                MAIN 590
C       6-10              INDK=0 FOR ISOTROPIC, INDK=1 FOR ORTHOTROPIC   MAIN 600
C      11-20              YOUNG'S MODULUS (ISOTROPIC MATERIAL)           MAIN 610
C                         E(L) (ORTHOTROPIC MATERIAL)                    MAIN 620
C      21-30              POISSON'S RATIO (ISOTROPIC MATERIAL)           MAIN 630
C                         E(T) (ORTHOTROPIC MATERIAL)                    MAIN 640
C      31-40              NOT USED (ISOTROPIC MATERIAL)                  MAIN 650
C                         NU(LT) (ORTHOTROPIC MATERIAL)                  MAIN 660
C      41-50              NOT USED (ISOTROPIC MATERIAL)                  MAIN 670
C                         NU(TL) (ORTHOTROPIC MATERIAL)                  MAIN 680
C              FOR ORTHOTROPIC MATERIALS, ONLY 3 OF THE ABOVE 4 ENGIN.   MAIN 690
C              CONSTANTS (E(L),E(T),NU(LT),NU(TL)) NEED BE KNOWN. FOR ANMAIN 700
C              UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.    MAIN 710
C      51-60              NOT USED (ISOTROPIC MATERIALS)                 MAIN 720
C                         G(LT) (ORTHOTROPIC MATERIALS)                  MAIN 730
C      61-70              ALPHA   COEFF. OF ISOTROPIC THERMAL EXPANSION  MAIN 740
C                         NOT USED FOR THERMALLY ANISOTROPIC MATERIALS   MAIN 750
C      71-80              MATERIAL THICKNESS. THIS IS USED FOR ELEMENTS  MAIN 760
C                         WHOSE PROPERTIES ARE NOT TO BE CALCULATED FROM MAIN 770
C                         INPUT LAMINA PROPERTIES.                       MAIN 780
C                                                                        MAIN 790
C     ELEMENT CARDS            (15I4,F10.5,I5)    NEL CARDS PER PROBLEM  MAIN 800
C       1-4                ELEMENT IDENTIFICATION NUMBER                 MAIN 810
C       5-52               ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)      MAIN 820
C      53-56              ELEMENT KIND - USE                             MAIN 830
C                          1  FOR A  FOUR  NODAL POINT ELEMENT           MAIN 840
C                          2  FOR AN EIGHT NODAL POINT ELEMENT           MAIN 850
C                          3  FOR A TWELVE NODAL POINT ELEMENT           MAIN 860
C      57-60              MATERIAL/LAMINATE IDENTIFICATION NUMBER        MAIN 870
C      61-70              THETA (DEGREES) USED FOR ORTHOTROPIC MATERIALS MAIN 880
C                        THETA IS THE ANGLE FROM THE PROBLEM BASED  X AXISMAIN 890
C                         TO  THE MATERIAL BASED L AXIS                  MAIN 900
C      71-75              ENTER 1 FOR A LAMINATED ELEMENT WHOSE          MAIN 910
C                         PROPERTIES ARE TO BE CALCULATED FROM           MAIN 920
```

```
C                     INPUT LAMINA PROPERTIES.                                MAIN 930
C                                                                             MAIN 940
C   BOUNDARY CONDITION CARDS  (2I5)           NPBC  CARDS PER PROBLEM         MAIN 950
C    COL.   VARIABLE                                                          MAIN 960
C     1-5  JOINT NUMBER                                                       MAIN 970
C     6-10 CODE   1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO     MAIN 980
C                 2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO     MAIN 990
C                 3 FOR A FIXED JOINT, BOTH DISP. COMPONENTS ARE ZERO         MAIN1000
C                 4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT.     MAIN1010
C                   IS SPECIFIED ON THE JOINT CARD IN COL. 41-50              MAIN1020
C                 5 FOR A ROLLER ON A Y AXIS, THE DISP. IN THE X DIRECT.      MAIN1030
C                   IS SPECIFIED ON THE JOINT CARD IN COL. 31-40              MAIN1040
C                 6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y          MAIN1050
C                   DIRECT. ON THE JOINT CARD IN COL.31-40 AND COL.41-50      MAIN1060
C                 7 FOR A ROLLER ON AN AXIS INCLINED ALPHA DEGREES            MAIN1070
C                   WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY            MAIN1080
C                   CONTAIN A TANGENTIAL LOAD IN COL. 31-40 AND THE           MAIN1090
C                   ANGLE ALPHA DEGREES IN COL. 41-50                         MAIN1100
C                                                                             MAIN1110
C    LAMINATE CARD DECK       AT LEAST TWO CARDS PER LAMINATE TYPE            MAIN1120
C                       USED TO CALCULATE THE PROPERTIES OF                   MAIN1130
C                       A LAMINATE FROM THE PROPERTIES OF                     MAIN1140
C                       THE INDIVIDUAL LAMINAE.                               MAIN1150
C    CARD ONE         2(I5)                                                   MAIN1160
C      COLUMN        VARIABLE                                                 MAIN1170
C        1-5         LAMTP        LAMINATE TYPE NUMBER (MAX.50)               MAIN1180
C        6-10        LYRNO        NO. LAMINAE IN THIS LAMTP (MAX.100)         MAIN1190
C    CARD TWO       4(I3,F8.3,F9.4)                                           MAIN1200
C      COLUMN        VARIABLE                                                 MAIN1210
C        1-3         MATERIAL NUMBER                                          MAIN1220
C        4-11        LAMINA THICKNESS                                         MAIN1230
C       12-20        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT       MAIN1240
C       21-23        MATERIAL NUMBER                                          MAIN1250
C       24-31        LAMINA THICKNESS                                         MAIN1260
C       32-40        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT       MAIN1270
C       41-43        MATERIAL NUMBER                                          MAIN1280
C       44-51        LAMINA THICKNESS                                         MAIN1290
C       52-60        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT       MAIN1300
C       61-63        MATERIAL NUMBER                                          MAIN1310
C       64-71        LAMINA THICKNESS                                         MAIN1320
C       72-80        LAMINA ORIENTATION (DEG) REL TO  L AXIS OF ELEMENT       MAIN1330
C    AS MANY OF THESE SECOND TYPE CARDS AS NEEDED TO SPECIFY LYRNO LAYERSMAIN1340
C    MAY BE USED                                                              MAIN1350
C    REPEAT THE SEQUENCE WITH NEW CARD TYPE 1 AND 2 AS NEEDED TO              MAIN1360
C    SPECIFY ALL  NLEL LAMINATE TYPES                                         MAIN1370
C   CONSISTENT LOAD VECTOR INFORMATION   (I5,5F15.0)  NJT CARDS               MAIN1380
C    COL.   VARIABLE                                                          MAIN1390
C     1-5       JOINT NUMBER                                                  MAIN1400
```

```
C     6-20       TEMPERATURE AT THE NODE                                       MAIN1410
C    21-35       X COMP. OF BODY FORCE INTENSITY AT THE NODE                   MAIN1420
C    36-50       Y COMP. OF BODY FORCE INTENSITY AT THE NODE                   MAIN1430
C    51-65       X COMP. OF INITIAL DISP. AT THE NODE                          MAIN1440
C    66-80       Y COMP. OF INITIAL DISP. AT THE NODE                          MAIN1450
C                                                                              MAIN1460
C   AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF       MAIN1470
C   A RUN MUST CONTAIN THE WORD    STOP     IN THE FIRST FOUR COLUMNS          MAIN1480
C                                                                              MAIN1490
      IMPLICIT REAL*8(A-H,O-Z)                                                 MAIN1500
      IMPLICIT INTEGER*2(I-N)                                                  MAIN1510
      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRMAIN1520
     1ES,NGP,LM(12),LJT(12),NLEL                                               MAIN1530
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR                                  MAIN1540
      COMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNPMAIN1550
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)                                  MAIN1560
      COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBYMAIN1570
     1,NTIN,NCARD                                                              MAIN1580
      COMMON /CC/ COORD(12,2),ELAST(3,3)                                       MAIN1590
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)                          MAIN1600
      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)                               MAIN1610
      DIMENSION FVEC(24), CLOAD(217,2)                                         MAIN1620
      PI = 3.14159265359D0                                                     MAIN1630
      MEL = 182                                                                MAIN1640
      MJT = 217                                                                MAIN1650
      MMT = 10                                                                 MAIN1660
      MBD = 86                                                                 MAIN1670
      MLEL = 50                                                                MAIN1680
      MLYR = 100                                                               MAIN1690
      NZRO = 0                                                                 MAIN1700
      ZRO = 0.0D0                                                              MAIN1710
    1 CALL INPUT                                                               MAIN1720
C                                                                              MAIN1730
      DO 2 I=1,MJT                                                             MAIN1740
C                                                                              MAIN1750
      DO 2 J=1,2                                                               MAIN1760
    2 CLOAD(I,J) = ZRO                                                         MAIN1770
C                                                                              MAIN1780
      NN = NSTRES                                                              MAIN1790
      NEQ = 2*NJT                                                              MAIN1800
C                                                                              MAIN1810
      DO 15 I=1,NEL                                                            MAIN1820
      NPEL = NCON(I,13)*4                                                      MAIN1830
      NJPE = NPEL*2                                                            MAIN1840
      INDIC = NCON(I,15)                                                       MAIN1850
      IF (INDIC.NE.1) GO TO 3                                                  MAIN1860
      CALL LAMC (I)                                                            MAIN1870
      GO TO 5                                                                  MAIN1880
```

```
    3 IMAT = NCON(I,14)
      THETA = ANGL(I)
      INDIC = INDK(IMAT)
      CALL STFMAT (IMAT,THETA,INDIC,NSTRES)
      IF (INDIC.NE.0) GO TO 4
      ALPHA = ELCON(IMAT,6)
      IF (NN.NE.0) ALPHA = ALPHA*(1.0D0+ELCON(IMAT,2))
    4 CONTINUE
      TK = ELCON(IMAT,7)
      IF (NSTRES.EQ.1) TK=1.0D0
    5 CONTINUE
C
      DO 6 J=1,NPEL
      IJ = NCON(I,J)
      LM(J) = IJ
      TEMP(J) = THNP(IJ)
C
      DO 6 JJ=1,2
      BCFY(J,JJ) = BYNP(IJ,JJ)
      DSPN(J,JJ) = DPIN(IJ,JJ)
    6 COORD(J,JJ) = COARD(IJ,JJ)
C
      IF (NTLD.EQ.0) GO TO 9
      LCODE = 1
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)
      WRITE (6,23) I,NTLD,NZRO,NZRO
C
      DO 7 L=1,NPEL
      IO = 2*L-1
      IE = IO+1
      J = NCON(I,L)
    7 WRITE (6,24) J,FVEC(IO),FVEC(IE)
C
C
      DO 8 J=1,NPEL
      IJ = LM(J)
      IO = 2*J-1
      IE = IO+1
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
    8 CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
C
    9 IF (NTBY.EQ.0) GO TO 12
      LCODE = 2
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)
      WRITE (6,23) I,NZRO,NTBY,NZRO
C
      DO 10 L=1,NPEL
      IO = 2*L-1
```

```
      IE = IO+1
      J = NCON(I,L)
   10 WRITE (6,24) J,FVEC(IO),FVEC(IE)
C
C
      DO 11 J=1,NPEL
      IJ = LM(J)
      IO = 2*J-1
      IE = IO+1
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
   11 CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
C
   12 IF (NTIN.EQ.0) GO TO 15
      LCODE = 3
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)
      WRITE (6,23) I,NZRO,NZRO,NTIN
C
      DO 13 L=1,NPEL
      IC = 2*L-1
      IE = IO+1
      J = NCON(I,L)
   13 WRITE (6,24) J,FVEC(IO),FVEC(IE)
C
C
      DC 14 J=1,NPEL
      IJ = LM(J)
      IO = 2*J-1
      IE = IO+1
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
   14 CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
C
   15 CCNTINUE
C
      KJT = NJT
C
      DC 19 I=1,NPBC
      NCODE = NBC(I,2)
      NJT = NBC(I,1)
      IF ((NCODE.EQ.1).OR.(NCODE.EQ.4)) CLOAD(NJT,2)=ZRO
      IF ((NCODE.EQ.2).OR.(NCODE.EQ.5)) CLOAD(NJT,1)=ZRO
      IF ((NCODE.EQ.3).OR.(NCODE.EQ.6)) GO TO 16
      GO TO 17
   16 CLOAD(NJT,1) = ZRO
      CLCAD(NJT,2) = ZRO
   17 IF (NCODE.EQ.7) GO TO 18
      GO TO 19
   18 ALF = TLOAD(NJT,2)*PI/180.0D0
      COSA = DCOS(ALF)
```

```
      SINA = DSIN(ALF)
      CLCAD(NJT,1) = CLOAD(NJT,1)*COSA+CLOAD(NJT,2)*SINA
      CLCAD(NJT,2) = ZRO
   19 CCNTINUE
C
      NJT = KJT
      WRITE (6,22)
C
      DC 20 I=1,NJT
      IF (NCARD.NE.0) WRITE (7,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)
      WRITE (6,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)
   20 CCNTINUE
C
      GC TO 1
C
   21 FORMAT (5X,1I5,2G25.16,1G15.8)
   22 FORMAT (//,10X,' RESULTANT  LOAD  VECTOR   (NTLD+NTBY+NTIN)',/,
     1  4X,' JOINT       X  COMPONENT',12X,' Y  COMPONENT',10X,'NODAL TEM
     2P.'//)
   23 FORMAT (//,5X,'ELEMENT',1I3,'    LOAD VECTOR',3I3,/,5X,'JOINT',7X,'
     1FX',13X,'FY',//)
   24 FORMAT (5X,1I5,2G15.6)
      END
      SUBROUTINE INPUT
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
     1ES,NGP,LM(12),LJT(12),NLEL
      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),THNP
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)
      CCMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBY
     1,NTIN,NCARD
      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
      CCMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
      DATA CHK/'STOP    '/
      PI = 3.14159265359D0
      ZRC = 0.0D0
      READ (5,25) TITLE
      WRITE (6,26) TITLE
      IF (TITLE(1).EQ.CHK) STOP
C
      DC 1 I=1,MJT
      THNP(I) = ZRO
C
      DO 1 J=1,2
      COARD(I,J) = ZRO
      CLCAD(I,J) = ZRO
```

```
    1 NBC(I,J) = 0
C
C
      DC 2 I=1,MMT
      INDK(I) = 0
C
      DC 2 J=1,7
    2 ELCON(I,J) = ZRO
C
C
      DO 3 I=1,MEL
      ANGL(I) = ZRO
C
      CC 3 J=1,15
    3 NCON(I,J) = 0
C
C
      DO 4 I=1,MLEL
      LYRNO(I) = 0
C
      DO 4 J=1,MLYR
      LAMAT(I,J) = 0
      CRLAM(I,J) = ZRO
    4 TKLAM(I,J) = ZRO
C
C
C     READ PROBLEM PARAMETERS
C
      READ (5,27) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NCA
     1RD,NLEL
      WRITE (6,28) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NC
     1ARD,NLEL
C
C     READ COORDINATES OF JCINTS AND CONCENTRATED LOADS
C
      WRITE (6,29)
C
      DC 5 I=1,NJT
      READ (5,30) IJT,COARD(IJT,1),COARD(IJT,2),CLOAD(IJT,1),CLOAD(IJT,2
     1),IND
      IF (IND.EQ.0) GO TO 5
      ALF = COARD(IJT,2)*PI/180.0D0
      CCSA = DCOS(ALF)
      SINA = DSIN(ALF)
      XC = COARD(IJT,1)*COSA
      YC = COARD(IJT,1)*SINA
      COARD(IJT,1) = XC
      CCARD(IJT,2) = YC
```

```
    5 CONTINUE
C
C
      DO 6 I=1,NJT
    6 WRITE (6,31) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)
C
C
C     READ MATERIAL PROPERTIES CARDS
C
C
      DO 7 I=1,NMAT
    7 READ (5,32) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)
C
      WRITE (6,33)
C
      DO 8 I=1,NMAT
      WRITE (6,34) I,(ELCON(I,J),J=1,7),INDK(I)
    8 CONTINUE
C
C
C     READ THE ELEMENT CARDS
C
      WRITE (6,35)
      WRITE (6,36)
C
      DO 9 I=1,NEL
    9 READ (5,37) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)
C
C
      DO 10 I=1,NEL
   10 WRITE (6,38) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)
C
C
C     READ BOUNDARY CONDITION CARDS
C
      WRITE (6,39)
C
      DO 11 I=1,NPBC
      READ (5,40) (NBC(I,J),J=1,2)
   11 WRITE (6,41) (NBC(I,J),J=1,2)
C
C
      DO 12 I=1,NPBC
C
      DO 12 J=I,NPBC
      IF (NBC(I,1).LE.NBC(J,1)) GO TO 12
      NT1 = NBC(I,1)
      NT2 = NBC(I,2)
```

```
      NBC(I,1) = NBC(J,1)
      NBC(I,2) = NBC(J,2)
      NBC(J,1) = NT1
      NBC(J,2) = NT2
   12 CCNTINUE
C
      NSV = 0
C
      DO 13 I=1,NPBC
      IF (NBC(I,2).NE.7) GO TO 13
      NSV = NSV+1
   13 CCNTINUE
C
      NPBCM = NPBC-1
      IF (NSV.EQ.0) GO TO 18
C
      DC 16 I=1,NSV
C
      DO 15 J=1,NPBCM
      IF (NBC(J,2).NE.7) GO TO 15
      NT1 = NBC(J,1)
C
      DO 14 K=J,NPBCM
      KP = K+1
      NBC(K,1) = NBC(KP,1)
   14 NBC(K,2) = NBC(KP,2)
C
      NBC(NPBC,1) = NT1
      NBC(NPBC,2) = 7
      GO TO 16
   15 CONTINUE
C
   16 CONTINUE
C
      WRITE (6,39)
C
      DC 17 I=1,NPBC
   17 WRITE (6,41) (NBC(I,J),J=1,2)
C
   18 CCNTINUE
      IF (NLEL.EQ.0) GO TO 21
C
C     READ LAMINATE CARD DECK
C
C
      DO 19 I=1,NLEL
      READ (5,42) LAMTP,LYRNO(LAMTP)
      L = LYRNO(LAMTP)
```

```
      READ (5,43) ((LAMAT(LAMTP,J),TKLAM(LAMTP,J),ORLAM(LAMTP,J)),J=1,L)INPT1700
   19 CONTINUE                                                          INPT1710
C                                                                       INPT1720
      WRITE (6,44)                                                      INPT1730
C                                                                       INPT1740
      DO 20 I=1,NLEL                                                    INPT1750
      WRITE (6,45) I,LYRNO(I)                                           INPT1760
      LL = LYRNO(I)                                                     INPT1770
C                                                                       INPT1780
      DO 20 J=1,LL                                                      INPT1790
   20 WRITE (6,46) J,LAMAT(I,J),TKLAM(I,J),ORLAM(I,J)                   INPT1800
C                                                                       INPT1810
   21 NLDC = NTLD+NTBY+NTIN                                             INPT1820
      IF (NLDC.EQ.0) GO TO 24                                           INPT1830
C                                                                       INPT1840
C     READ CONSISTENT LOAD VECTOR INFORMATION                           INPT1850
C                                                                       INPT1860
      WRITE (6,47)                                                      INPT1870
C                                                                       INPT1880
      DO 22 I=1,NJT                                                     INPT1890
   22 READ (5,48) II,THNP(II),BYNP(II,1),BYNP(II,2),DPIN(II,1),DPIN(II,2INPT1900
     1)                                                                 INPT1910
C                                                                       INPT1920
C                                                                       INPT1930
      DO 23 I=1,NJT                                                     INPT1940
   23 WRITE (6,49) I,THNP(I),BYNP(I,1),BYNP(I,2),DPIN(I,1),DPIN(I,2)    INPT1950
C                                                                       INPT1960
      RETURN                                                            INPT1970
   24 WRITE (6,50)                                                      INPT1980
      STOP                                                              INPT1990
C                                                                       INPT2000
   25 FORMAT (10A8)                                                     INPT2010
   26 FORMAT (1X,10A8)                                                  INPT2020
   27 FORMAT (12I5)                                                     INPT2030
   28 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TOTAL NUMBER OF JOINTS='INPT2040
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOADINPT2050
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' THINPT2060
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS POINTS USED ',I5,     INPT2070
     4//,' THERMAL LOAD      ',I5,//,' BODY FORCE LOAD    ',I5,//,      INPT2080
     5' INITIAL STRESS LOAD      ',I5,//,' NCARD',I5,//,'NUMBER OF LAMINATINPT2090
     6E TYPES USED',I5)                                                 INPT2100
   29 FORMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7XINPT2110
     1,'X LOAD',9X,' Y LOAD'//)                                         INPT2120
   30 FORMAT (I5,5X,4G15.8,I5)                                          INPT2130
   31 FORMAT (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)               INPT2140
   32 FORMAT (2I5,7F10.2)                                               INPT2150
   33 FORMAT (///,' MATERIAL PROPERTIES',//,1X,'MATERIAL     ',2X,'E,E(L)INPT2160
     1',7X,'NU,E(LT)',5X,'NU(LT)',8X,'NU(TL)',10X,'G(LT)',12X,'ALPHA',9XINPT2170
```

```
  2,'THICKNESS',2X,'INDK')
34 FORMAT (' ',1X,I3,5X,7(2X,G13.6),2X,I5)
35 FORMAT (///,' ELEMENT CARDS - CONNECTIVITY MATRIX')
36 FORMAT (//,' ELEMENT NUMBER',20X,'JOINT NUMBERS',30X,'KIND',3X,'MA
  1TERIAL',3X,'THETA(DEG.)',3X,'LAM. IND.')
37 FORMAT (15I4,F10.5,I5)
38 FORMAT (6X,I3,6X,12I5,1X,I5,5X,I5,1X,F10.5,5X,I5)
39 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)
40 FORMAT (2I5)
41 FORMAT (5X,I5,I10)
42 FORMAT (2I5)
43 FORMAT (4(I3,F8.3,F9.4))
44 FORMAT (///,'LAMINATE PROPERTIES',//,1X,'LAMINATE TYPE',5X,'NO OF
  1LAYERS    ',1X,'LAYER NUMBER',5X,'MATERIAL NO.',5X,'THICKNESS',5X,'
  2 ORIENTATION'//)
45 FORMAT (4X,I5,11X,I5)
46 FORMAT (30X,I5,8X,I7,10X,F9.3,5X,F9.3)
47 FORMAT (//,' DISTRIBUTED LOAD INFORMATION ',//,
  1          24X,' BODY FORCE          INITIAL  DISP.',/,2X,
  2 ' JOINT      TEMP.    X COMP.    Y COMP.    X COMP.    Y COMP.',//)
48 FORMAT (1I5,5F15.0)
49 FORMAT (4X,1I3,3X,5G10.3)
50 FORMAT (///,5X,' *** NO DISTRIBUTED LOAD *** ')
   END
   SUBROUTINE LAMC (IK)
C   THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED
C   COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED
C   COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING
C   PROPERTIES ARE UNCOUPLED.IT CALCULATES AN AVERAGE STIFFNESS MATRIX
C   FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.
C   IK IS THE ELEMENT NUMBER
   IMPLICIT REAL*8(A-H,O-Z)
   IMPLICIT INTEGER*2(I-N)
   COMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNP
  1(217),BYNP(217,2),DPIN(217,2),ANGL(182)
   COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
  1ES,NGP,LM(12),LJT(12),NLEL
   COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBY
  1,NTIN,NCARD
   COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
   COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
   COMMON /CC/ COORD(12,2),ELAST(3,3)
   DIMENSION C(3,3)
   LAMTP = NCON(IK,14)
   THETA = ANGL(IK)
   TK = 0.D0
C
   DO 1 I=1,3
```

```
C
      DC 1 J=1,3
    1 C(I,J) = 0.DO
C
      LYRS = LYRNO(LAMTP)
C
      DO 3 I=1,LYRS
      MATN = LAMAT(LAMTP,I)
      PHI = THETA+ORLAM(LAMTP,I)
      INDIC = INDK(MATN)
      NSTRES = NN
      CALL STFMAT (MATN,PHI,INDIC,NSTRES)
      TK = TK+TKLAM(LAMTP,I)
C
      DC 2 J=1,3
C
      DC 2 K=1,3
    2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)
C
    3 CCNTINUE
C
C
      DO 4 J=1,3
C
      DC 4 K=1,3
    4 ELAST(J,K) = C(J,K)/TK
C
      RETURN
      END
      SUBROUTINE STFMAT (MATN,THETA,INDIC,NSTRES)
C   THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS MATRIX FOR A HOOKEAN
C   ELEMENT.THE SUBROUTINE CALCULATES AND STORES IN C(3X3)THE VALUES
C   OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER
C   EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS
C   EXHIBITING TWO DIMENSIONAL ORTHOTROPY.FOR ANISCTROPIC MATERIALS,
C   THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL
C   BASED L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM
C   BASED X-Y COORDINATE SYSTEM.
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CCMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNP
     1(217),BYNP(217,2),DPIN(217,2),ANGL(182)
      CCMMON /CC/ COORD(12,2),C(3,3)
      DIMENSION A(3,3)
C
      DC 1 I=1,3
C
      DC 1 J=1,3
```

```
      A(I,J) = 0.0D0
      C(I,J) = 0.0D0
    1 CONTINUE
C
      IF (INDIC.EQ.1) GO TO 3
      IF (NSTRES.EQ.1) GO TO 2
C    FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS
C
      E = ELCON(MATN,1)
      XNU = ELCON(MATN,2)
      E0 = E/(1.0D0-XNU*XNU)
      C(1,1) = E0
      C(1,2) = E0*XNU
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,3) = E0*(1.0D0-XNU)/2.0D0
      RETURN
    2 CONTINUE
C
C   FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN
C
      E = ELCON(MATN,1)
      XNU = ELCON(MATN,2)
      E1 = E/((1.0D0+XNU)*(1.0D0-2.0D0*XNU))
      C(1,1) = E1*(1.0D0-XNU)
      C(1,2) = E1*XNU
      C(2,1) = C(1,2)
      C(2,2) = C(1,1)
      C(3,3) = E1*(0.5D0-XNU)
      RETURN
    3 CONTINUE
C
C   FORM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASED
C   L-T COORDINATE SYSTEM
C
      EL = ELCON(MATN,1)
      ET = ELCON(MATN,2)
      XNULT = ELCON(MATN,3)
      XNUTL = ELCON(MATN,4)
      GLT = ELCON(MATN,5)
      IF (EL.LT.-0.0D0) EL=XNULT*ET/XNUTL
      IF (ET.LT.-0.0D0) ET=EL*XNUTL/XNULT
      IF (XNULT.LT.-0.0D0) XNULT=EL*XNUTL/ET
      IF (XNUTL.LT.-0.0D0) XNUTL=XNULT*ET/EL
      Z = 1.0D0-XNULT*XNUTL
      A(1,1) = EL/Z
      A(1,2) = ET*XNULT/Z
      A(2,1) = A(1,2)
```

```
      A(2,2) = ET/Z
      A(3,3) = GLT
C
C    THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE
C    SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM.THE TRANSFORMATION IS
C    ACCCMPLISHED BY ROTATION ABOUT THE Z AXIS THRCUGH THETA DEGREES.
C
      PI = 3.14159265359D0
      THRAD = THETA*PI/180.0D0
      CCS1 = DCOS(THRAD)
      CCS2 = DCOS(THRAD)**2
      COS3 = DCOS(THRAD)**3
      COS4 = DCOS(THRAD)**4
      SIN1 = DSIN(THRAD)
      SIN2 = DSIN(THRAD)**2
      SIN3 = DSIN(THRAD)**3
      SIN4 = DSIN(THRAD)**4
      TWO = 2.0D0
      FOUR = 4.0D0
      C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4
      C(1,2) = (A(1,1)+A(2,2)-FOUR*A(3,3))*SIN2*COS2+A(1,2)*(SIN4+COS4)
      C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*(
     1A(3,3)))*SIN3*COS1
      C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4
      C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*A
     1(3,3))*SIN1*COS3
      C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3)))*SIN2*COS2+A(3,3)*(SIN
     14+COS4)
      C(2,1) = C(1,2)
      C(3,1) = C(1,3)
      C(3,2) = C(2,3)
      RETURN
      END
      SUBROUTINE FORMB (B,X,Y,N,DTJ,IND)
C
C THIS SUBROUTINE FORMS THE   B   MATRIX AS A FUNCTION OF
C XI AND ETA FOR THE THREE DIFFERENT QUADRILATERAL ELEM. IN THIS FAMILY
C
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24)
      CCMMON /CC/ COORD(12,2),ELAST(3,3)
      ZERO = 0.0D0
C
      DO 1 I=1,3
C
      DO 1 J=1,N
    1 B(I,J) = ZERO
```

```
C
      ONE = 1.0D0
      TWO = 2.0D0
      FCUR = 4.0D0
      NPT = N/2
C
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA
C
      IGO = N/8
      GO TO (2,3,4), IGO
C
C     LINEAR FUNCTIONS
C
    2 W1(1,3) = -(ONE-Y)/FOUR
      W1(1,4) = -W1(1,3)
      W1(1,1) = (ONE+Y)/FOUR
      W1(1,2) = -W1(1,1)
      W1(2,3) = -(ONE-X)/FOUR
      W1(2,4) = -(ONE+X)/FOUR
      W1(2,1) = -W1(2,4)
      W1(2,2) = -W1(2,3)
      GO TO 5
    3 CCNTINUE
C
C    GUACRATIC FUNCTIONS
C
      TXPY = TWO*X+Y
      TXMY = TWO*X-Y
      TYPX = TWO*Y+X
      TYMX = TWO*Y-X
      OMY = ONE-Y
      OPY = ONE+Y
      OPX = ONE+X
      OMX = ONE-X
      W1(1,5) = OMY*TXPY/FOUR
      W1(1,6) = -OMY*X
      W1(1,7) = OMY*TXMY/FOUR
      W1(1,8) = OPY*OMY/TWO
      W1(1,1) = OPY*TXPY/FOUR
      W1(1,2) = -OPY*X
      W1(1,3) = OPY*TXMY/FOUR
      W1(1,4) = -OPY*OMY/TWO
      W1(2,5) = OMX*TYPX/FOUR
      W1(2,6) = -OPX*OMX/TWO
      W1(2,7) = OPX*TYMX/FOUR
      W1(2,8) = -Y*OPX
      W1(2,1) = OPX*TYPX/FOUR
      W1(2,2) = OPX*OMX/TWO
```

```
      W1(2,3) = OMX*TYMX/FOUR
      W1(2,4) = -Y*OMX
      GO TO 5
    4 CONTINUE
C
C     CUBIC FUNCTIONS
C
      OPX = ONE+X
      OPY = ONE+Y
      OMX = ONE-X
      OMY = ONE-Y
      TPTMNX = 3.0D0+2.0D0*X-9.0D0*X*X
      TMTMNX = 3.0D0-2.0D0*X-9.0D0*X*X
      TPTMNY = 3.0D0+2.0D0*Y-9.0D0*Y*Y
      TMTMNY = 3.0D0-2.0D0*Y-9.0D0*Y*Y
      TYPO = 3.0D0*Y+ONE
      TYMO = 3.0D0*Y-ONE
      TXPO = 3.0D0*X+ONE
      TXMO = 3.0D0*X-ONE
      TMTPEX = 10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y
      TMTMEX = 10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y
      TMNPEY = 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y
      TMNMEY = 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y
      TTT = 32.0D0
      TTN = 32.0D0/9.0D0
      W1(1,7) = OMY*TMTPEX/TTT
      W1(1,8) = -TPTMNX*OMY/TTN
      W1(1,9) = TMTMNX*OMY/TTN
      W1(1,10) = -OMY*TMTMEX/TTT
      W1(1,11) = -OPY*OMY*TYMO/TTN
      W1(1,12) = OPY*OMY*TYPO/TTN
      W1(1,1) = -OPY*TMTMEX/TTT
      W1(1,2) = TMTMNX*OPY/TTN
      W1(1,3) = -TPTMNX*OPY/TTN
      W1(1,4) = OPY*TMTPEX/TTT
      W1(1,5) = -OPY*OMY*TYPO/TTN
      W1(1,6) = OPY*OMY*TYMO/TTN
      W1(2,7) = OMX*TMNPEY/TTT
      W1(2,8) = OPX*OMX*TXMO/TTN
      W1(2,9) = -OPX*OMX*TXPO/TTN
      W1(2,10) = OPX*TMNPEY/TTT
      W1(2,11) = -TPTMNY*OPX/TTN
      W1(2,12) = TMTMNY*OPX/TTN
      W1(2,1) = -OPX*TMNMEY/TTT
      W1(2,2) = OPX*OMX*TXPO/TTN
      W1(2,3) = -OPX*OMX*TXMO/TTN
      W1(2,4) = -OMX*TMNMEY/TTT
      W1(2,5) = OMX*TMTMNY/TTN
```

```
      W1(2,6)  =  -OMX*TPTMNY/TTN                                      FRMB1120
    5 CONTINUE                                                          FRMB1130
C                                                                       FRMB1140
C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)                        FRMB1150
C                                                                       FRMB1160
C                                                                       FRMB1170
      DO 6 I=1,2                                                        FRMB1180
C                                                                       FRMB1190
      DO 6 J=1,2                                                        FRMB1200
      AJ(I,J)  =  ZERO                                                  FRMB1210
C                                                                       FRMB1220
      DC 6 K=1,NPT                                                      FRMB1230
    6 AJ(I,J)  =  AJ(I,J)+W1(I,K)*COORD(K,J)                            FRMB1240
C                                                                       FRMB1250
C                                                                       FRMB1260
C CALCULATE DETERMINANT OF THE JACOBIAN                                 FRMB1270
C                                                                       FRMB1280
      DTJ  =  AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)                           FRMB1290
      IF (IND.NE.0) RETURN                                              FRMB1300
C                                                                       FRMB1310
C INVERT JACOBIAN IN AJIN                                               FRMB1320
C                                                                       FRMB1330
      AJIN(1,1)  =  AJ(2,2)/DTJ                                         FRMB1340
      AJIN(1,2)  =  -AJ(1,2)/DTJ                                        FRMB1350
      AJIN(2,1)  =  -AJ(2,1)/DTJ                                        FRMB1360
      AJIN(2,2)  =  AJ(1,1)/DTJ                                         FRMB1370
C                                                                       FRMB1380
C FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y       FRMB1390
C                                                                       FRMB1400
C                                                                       FRMB1410
      DC 7 I=1,2                                                        FRMB1420
C                                                                       FRMB1430
      DO 7 J=1,NPT                                                      FRMB1440
      DNX(I,J)  =  ZERO                                                 FRMB1450
C                                                                       FRMB1460
      DO 7 K=1,2                                                        FRMB1470
    7 DNX(I,J)  =  DNX(I,J)+AJIN(I,K)*W1(K,J)                           FRMB1480
C                                                                       FRMB1490
C                                                                       FRMB1500
C FORM B(I,J) MATRIX                                                    FRMB1510
C                                                                       FRMB1520
C                                                                       FRMB1530
      DC 8 I=1,NPT                                                      FRMB1540
      ICD  =  2*I-1                                                     FRMB1550
      IEV  =  2*I                                                       FRMB1560
      B(1,IOD)  =  DNX(1,I)                                             FRMB1570
      B(2,IEV)  =  DNX(2,I)                                             FRMB1580
      B(3,IEV)  =  DNX(1,I)                                             FRMB1590
```

```
    8 B(3,IOD) = DNX(2,I)
C
      RETURN
      END
      SUBROUTINE FLOAD (FVEC,N,LCCDE,NGP)
C     FLOAD IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE
C     CONSISTENT LOAD VECTORS FOR ANY OF THE THREE ELEMENTS OF THIS
C     FAMILY OF ISOPARAMETRIC ELEMENTS
C     GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.
C     FVEC(24) IS THE OUTPUT ARRAY CONTAINING THE LOAD VECTOR UPON EXIT
C     N IS EQUAL TO TWICE THE NUMBER OF NODAL POINTS IN THE ELEMENT
C     LCCDE MUST BE EQUAL TO  1 FOR A TEMPERATURE LOAD,
C                             2 FOR A BODY FORCE LOAD VECTOR,
C                             3 FOR AN INITIAL DEFORMATION LOAD VECTOR.
C          NGP IS THE NUMBER OF GAUSS POINTS USED FOR INTEGRATION
C          COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL
C                      THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED
C          ELAST(3,3)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS
C                      OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS
C                      IS: SIGMAX,SIGMAY,TAUXY
C          TEMP(12) IS A VECTOR OF NODAL TEMPERATURES
C          BDFY(12,2) IS A VECTOR OF NODAL VALUES OF THE BODY FORCE
C          DSPN(12,2) IS A VECTOR OF INITIAL VALUES OF DISPLACEMENTS
C          TK IS THE THICKNESS OF THE ELEMENT ( UNITY FOR PLANE STRAIN)
C          ALPHA IS THE COEFFICIENT OF EXPANSION OF THE MATERIAL
C          NN IS EQUAL TO 1 FOR PLANE STRAIN,0 FOR PLANE STRESS
C          NTLD IS EQUAL TO  1 FOR A THERMAL LOAD,   0 OTHERWISE
C          NTBY IS EQUAL TO  1 FOR A BODY FORCE LOAD,0 OTHERWISE
C          NTIN IS EQUAL TO  1 FOR AN IN.STR. LOAD,  0 OTHERWISE
C          NCARD IS THE PUNCH CONTROL CARD
C     CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBY
     1,NTIN,NCARD
      COMMON /CC/ COORD(12,2),ELAST(3,3)
      DIMENSION FVEC(24), VAL(12), BTD(24,3), WVEC(24), SK(24,24), B(3,2
     14)
      DIMENSION DSPO(24)
      DIMENSION XI(5), AI(5), AIA(5,5)
      DIMENSION X2(2), X3(3), X4(4), X5(5), A2(2), A3(3), A4(4), A5(5)
      DATA X2/.57735026918962 6D0,-.57735026918962 6D0/
      DATA A2/1.0D0,1.0D0/
      DATA X3/.774596669241483D0,0.0D0,-.774596669241483D0/
      DATA A3/.555555555555556D0,.888888888888889D0,.555555555555556D0/
      DATA X4/.861136311594 0D0,.339981043584 8D0,-.339981043584 8D0,-.8611
     1363115940D0/
      DATA A4/.347854845137 4D0,.652145154862 5D0,.652145154862 5D0,.347854
```

```
      18451374D0/
       DATA X5/.906179845938 6D0,.5384693101056D0,0.0D0,-.5384693101056D0,
      1-.9061798459386D0/
       CATA A5/.2369268850561D0,.4786286704993D0,.5688888888888D0,.478628
      16704993D0,.2369268850561D0/
       IF (NGP.EQ.0) NGP = 5
       IF (TK.EQ.0.0D0) TK=1.0D0
C
       DC 1 I=1,24
       FVEC(I) = 0.0D0
     1 WVEC(I) = 0.0D0
C
       JUMP = NGP-1
C
       DC 6 I=1,NGP
       GO TO (2,3,4,5), JUMP
     2 XI(I) = X2(I)
       AI(I) = A2(I)
       GO TO 6
     3 XI(I) = X3(I)
       AI(I) = A3(I)
       GC TO 6
     4 XI(I) = X4(I)
       AI(I) = A4(I)
       GO TO 6
     5 XI(I) = X5(I)
       AI(I) = A5(I)
     6 CCNTINUE
C
C
       DC 7 I=1,NGP
C
       DO 7 J=1,NGP
     7 AIA(I,J) = AI(I)*AI(J)
C
       NPEL = N/2
C
       DC 24 I=1,NGP
       X = XI(I)
C
       DO 24 J=1,NGP
       Y = XI(J)
       GC TO (8,13,17), LCODE
     8 ILT = 0
       CALL FORMB (B,X,Y,N,DTJ,ILT)
C
       DO 9 II=1,N
C
```

```
      DO 9 JJ=1,3
      BTD(II,JJ) = 0.0D0
C
      DO 9 KK=1,3
    9 BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)
C
      CALL SHAPE (VAL,X,Y,NPEL)
      CFT = 0.0D0
C
      DC 10 II=1,NPEL
   10 CFT = CFT+VAL(II)*TEMP(II)
C
      CFT = CFT*ALPHA*TK*DTJ
C
      DO 11 II=1,N
   11 WVEC(II) = (BTD(II,1)+BTD(II,2))*CFT
C
C
      DC 12 II=1,N
   12 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)
C
      GC TO 24
   13 CALL SHAPE (VAL,X,Y,NPEL)
      ILT = 1
      CALL FORMB (B,X,Y,N,DTJ,ILT)
      CFTX = 0.0D0
      CFTY = 0.0D0
C
      DC 14 II=1,NPEL
      CFTX = CFTX+VAL(II)*BDFY(II,1)
   14 CFTY = CFTY+VAL(II)*BDFY(II,2)
C
C
      DC 15 II=1,NPEL
      IC = 2*II-1
      IE = IO+1
      WVEC(IO) = VAL(II)*CFTX*DTJ*TK
   15 WVEC(IE) = VAL(II)*CFTY*DTJ*TK
C
C
      DC 16 II=1,N
   16 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)
C
      GC TO 24
   17 ILT = 0
      CALL FORMB (B,X,Y,N,DTJ,ILT)
C
      DO 18 II=1,N
```

```
C                                                                       FLOD1410
      DO 18 JJ=1,3                                                      FLOD1420
      BTD(II,JJ) = 0.0D0                                                FLOD1430
C                                                                       FLOD1440
      DO 18 KK=1,3                                                      FLOD1450
   18 BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)                     FLOD1460
C                                                                       FLOD1470
C                                                                       FLOD1480
      DO 19 II=1,N                                                      FLOD1490
C                                                                       FLOD1500
      DO 19 JJ=1,N                                                      FLOD1510
      SK(II,JJ) = 0.0D0                                                 FLOD1520
C                                                                       FLOD1530
      DO 19 KK=1,3                                                      FLOD1540
   19 SK(II,JJ) = SK(II,JJ)+BTD(II,KK)*B(KK,JJ)                         FLOD1550
C                                                                       FLOD1560
C                                                                       FLOD1570
      DC 20 II=1,NPEL                                                   FLOD1580
      IC = 2*II-1                                                       FLOD1590
      IE = IO+1                                                         FLOD1600
      DSPO(IO) = DSPN(II,1)                                             FLOD1610
   20 DSPO(IE) = DSPN(II,2)                                             FLOD1620
C                                                                       FLOD1630
C                                                                       FLOD1640
      DO 21 II=1,N                                                      FLOD1650
      WVEC(II) = 0.0D0                                                  FLOD1660
C                                                                       FLOD1670
      DC 21 JJ=1,N                                                      FLOD1680
   21 WVEC(II) = WVEC(II)+SK(II,JJ)*DSPC(JJ)                            FLOD1690
C                                                                       FLOD1700
C                                                                       FLOD1710
      DC 22 II=1,N                                                      FLOD1720
   22 WVEC(II) = -WVEC(II)*TK*DTJ                                       FLOD1730
C                                                                       FLOD1740
C                                                                       FLOD1750
      DC 23 II=1,N                                                      FLOD1760
   23 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)                             FLOD1770
C                                                                       FLOD1780
   24 CCNTINUE                                                          FLOD1790
C                                                                       FLOD1800
      RETURN                                                            FLOD1810
      END                                                               FLOD1820
      SUBROUTINE SHAPE (VAL,X,Y,NPEL)                                   SHPE  10
      IMPLICIT REAL*8(A-H,O-Z)                                          SHPE  20
      IMPLICIT INTEGER*2(I-N)                                           SHPE  30
      DIMENSION VAL(12), XYL(4,2), XYQ(8,2), XYC(12,2), IPERM(4)        SHPE  40
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/     SHPE  50
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,SHPE  60
```

```
     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/
      DATA XYC/1.0D0,.33333333333333D0,-.33333333333333D0,-1.0D0,-1.0D0,-1
     1.0D0,-1.0D0,-.33333333333333D0,.33333333333333D0,1.0D0,1.0D0,1.0D0,1
     2.0D0,1.0D0,1.0D0,1.0D0,.33333333333333D0,-.33333333333333D0,-1.0D0,-
     31.0D0,-1.0D0,-1.0D0,-.33333333333333D0,.33333333333333D0/
      FL(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)/FOUR
      FQC(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)*(X*X1+Y*Y1-ONE)/FOUR
      FQM(X,Y,Y1) = (ONE-X*X)*(ONE+Y*Y1)/TWO
      FCC(X,Y,X1,Y1) = (ONE+X*X1)*(ONE+Y*Y1)*(.9D1*(X*X+Y*Y)-.1D2)/3.2D1
      FCM(X,Y,X1,Y1) = (ONE-X*X)*(ONE+.9D1*X*X1)*(ONE+Y*Y1)*.9D1/3.2D1
      CNE = 1.0D0
      TWO = 2.0D0
      FOUR = 4.0D0
      IGO = NPEL/4
      GO TO (1,3,7), IGO
    1 CONTINUE
C         LINEAR FUNCTIONS
C
      DO 2 I=1,4
      X1 = XYL(I,1)
      Y1 = XYL(I,2)
    2 VAL(I) = FL(X,Y,X1,Y1)
C
      RETURN
    3 CCNTINUE
C         QUACRATIC FUNCTIONS
C
      DO 4 I=1,7,2
      X1 = XYC(I,1)
      Y1 = XYQ(I,2)
    4 VAL(I) = FQC(X,Y,X1,Y1)
C
C
      DC 5 I=2,6,4
      Y1 = XYQ(I,2)
    5 VAL(I) = FQM(X,Y,Y1)
C
C
      DO 6 I=4,8,4
      X1 = XYQ(I,1)
    6 VAL(I) = FQM(Y,X,X1)
C
      RETURN
    7 CONTINUE
C         CUBIC FUNCTIONS
C
      DO 8 I=1,10,3
      X1 = XYC(I,1)
```

```
      Y1 = XYC(I,2)                                                     SHPE 550
    8 VAL(I) = FCC(X,Y,X1,Y1)                                           SHPE 560
C                                                                       SHPE 570
      IPERM(1) = 2                                                      SHPE 580
      IPERM(2) = 3                                                      SHPE 590
      IPERM(3) = 8                                                      SHPE 600
      IPERM(4) = 9                                                      SHPE 610
C                                                                       SHPE 620
      DO 9 I=1,4                                                        SHPE 630
      IJ = IPERM(I)                                                     SHPE 640
      X1 = XYC(IJ,1)                                                    SHPE 650
      Y1 = XYC(IJ,2)                                                    SHPE 660
    9 VAL(IJ) = FCM(X,Y,X1,Y1)                                          SHPE 670
C                                                                       SHPE 680
      IPERM(1) = 5                                                      SHPE 690
      IPERM(2) = 6                                                      SHPE 700
      IPERM(3) = 11                                                     SHPE 710
      IPERM(4) = 12                                                     SHPE 720
C                                                                       SHPE 730
      DO 10 I=1,4                                                       SHPE 740
      IJ = IPERM(I)                                                     SHPE 750
      X1 = XYC(IJ,1)                                                    SHPE 760
      Y1 = XYC(IJ,2)                                                    SHPE 770
   10 VAL(IJ) = FCM(Y,X,Y1,X1)                                          SHPE 780
C                                                                       SHPE 790
      RETURN                                                            SHPE 800
      END                                                               SHPE 810
```

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